MathsBites by Clifford the Dog

Factorials and prime numbers

People in a row

A tree diagram, for example www.techuser.net/randpermgen.html, can be used to illustrate how many different ways a group of people can be arranged in a row.

If there are 4 people they can be arranged in 4 factorial, written $4! = 4 \times 3 \times 2 \times 1 = 24$ different ways. The function $f(n) = n!$ calculates this for n people and can be evaluated as above, or recursively as follows:

$$f(1) = 1; f(n + 1) = (n + 1) f(n)$$

Thus $f(2) = 2 \times f(1) = 2 \times 1 = 2$;
$f(3) = 3 \times f(2) = 3 \times 2 = 6$;
$f(4) = 4 \times f(3) = 4 \times 6 = 24$ and so on.

Use this method to evaluate $f(n)$ for n from 5 to 10. What value should be assigned to $f(0)$?

http://en.wikipedia.org/wiki/File:Russian-Matroshka_no_bg.jpg (creative commons)

Activity: Six seats (a pair of three seats opposite each other) are available in a train carriage, and at the next stop three people come in and sit in them. How many different seating arrangements are possible?

Finding some more prime numbers

http://colintgraham.com/mathematics/activities-and-resources/101-manipulative-lessons-with-lego/

See link 17.

Any number greater than 1 generated by $f(n) = n!$ is definitely not a prime number as each of 1, 2 ... n is a factor. What about $f(n) + 1$, will it be prime? Sometimes it is, for example, $f(3) + 1 = 7$ is prime, but other times it is not, for example, $f(4) + 1 = 25$ is not prime. However 25 has a prime factor, 5, that lies between 4 and 25.

Construct a table of values for $f(n) + 1$ for n from 1 to 20 and identify either $f(n) + 1$ as prime or identify a prime factor of $f(n) + 1$ between n and $f(n) + 1$.

Activity: Explain why, for $n > 1$, either $f(n) + 1$ is prime, or that there is another prime number between n and $f(n) + 1$. To locate small primes see: http://en.wikipedia.org/wiki/Sieve_of_Eratosthenes which includes an interesting animation.