Strange Balances

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Comment

- I am happy to be contacted by email (dholton@unimelb.edu.au)
- It may take me a day or two to answer email as I don’t come to university every day.
- I am also happy to meet individuals or groups whenever or wherever it is mutually convenient.
- I’m happy to give anyone a copy of these slides

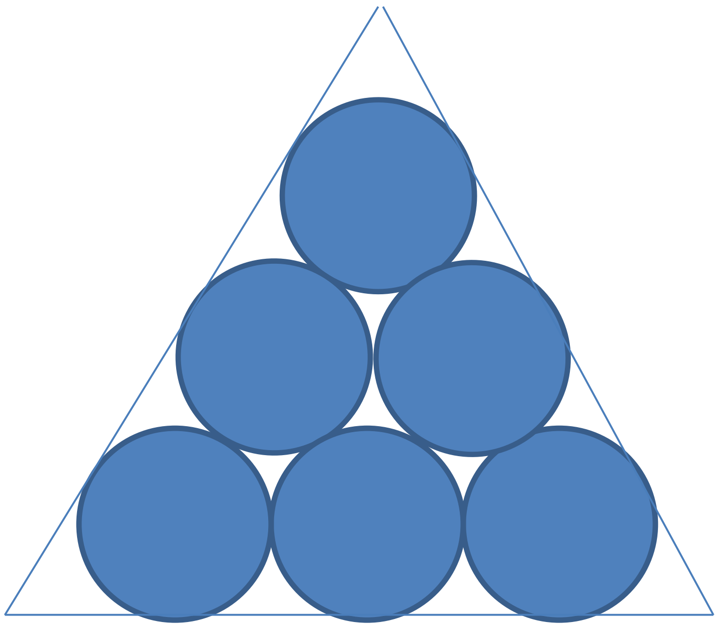
Aims

- To help you to make problems (and ***ANY MATHS***), **your own**
- To provide a problem (a recipe) that you can do with ***any*** class
- To provide sources for more such problems
- (To show how mathematics research is done)

The Problem I

Can you put ***all*** the weights 1gm, 2gm, 3gm, 4gm, 5gm, 6gm in the six circular spaces such that:

the ‘scales’ balance and only one weight is used in each space?



The Problem II

- Is the problem clear? Any questions?

- Do you think the problem has an answer? Hands up for yes/no/maybe.

  If it is ‘yes’, show a solution;

  if it’s ‘no’, prove that there is no solution;

  Otherwise you should probably play with the question for a while and then make a decision.

- Work with neighbours; when you have something to say, let me know.

The Problem III

What do your jottings tell you?

Do you have any *conjectures*?

Discuss with your neighbour.

Discuss with all of us.

The Problem IV

- How can you prove your conjecture?

  Is your proof correct?

  Does this lead to other conjectures?

  Discuss with your neighbours.

  Discuss with all of us.

Problem V

- Call one of these balance arrangements a ***thingie***.

A Proof – side sums

- Let's first look at the side sum of weights:

  Now 6 has to go somewhere; so the smallest sum we can make is with 1 and 2. So the smallest sum is 9.

  Now 1 has to go somewhere; so the largest sum we can make is with 5 and 6. So the largest sum is 12.

- So side sums can be 9, 10, 11, 12.

A proof – a thingie a side sum

- Let's look at the side sum of 9 (the others can be done similarly).

The only ways to make 9 using 1 to 6 are

$$9 = 1 + 2 + 6$$
$$= 1 + 3 + 5$$
$$= 2 + 3 + 4.$$

Why does this give us just one ***thingie***? Are you sure?

For the record

- What other thingies are there?

Use?

- What classes would you do this with?
- Why those grades/years?
- What would your aim be?
- What help would you expect to have to give them?

But what next?

- Can you ***extend*** this problem?

    find another problem like it

- Can you ***generalise*** this problem

    find a single problem that is based on this one but include this one and an infinite number of others

    e.g. Pythagoras Theorem

- What ***conjectures*** do you have on your ideas?

Discuss

- What ideas do you have?
- How could you tackle them?

My choice

- Take any six different weights. Call them **nice**
  if they can form a thingie

- Are there any sets of 6 weights that aren't nice?

- **Aim**: to find all nice sets

Nice sets

- Tell me some nice sets
- Can you find the most general nice set
- What does it look like?

Try this

- If I give you 1 number can you **always** find 5 others that make a nice set?
- If I give you 2 numbers can you **always** find 4 others that make a nice set?
- If I give you 3 numbers can you **always** find 3 others that make a nice set?
- If I give you 4 numbers can you **always** find 2 others that make a nice set?
- If I give you 5 numbers can you **always** find 1 other that make a nice set?
- Discuss and report back

Any general nice thingie?

- What nice sets do you have?

- Can you find the most general nice set?
  - What does it look like?
  - There is no such thing.

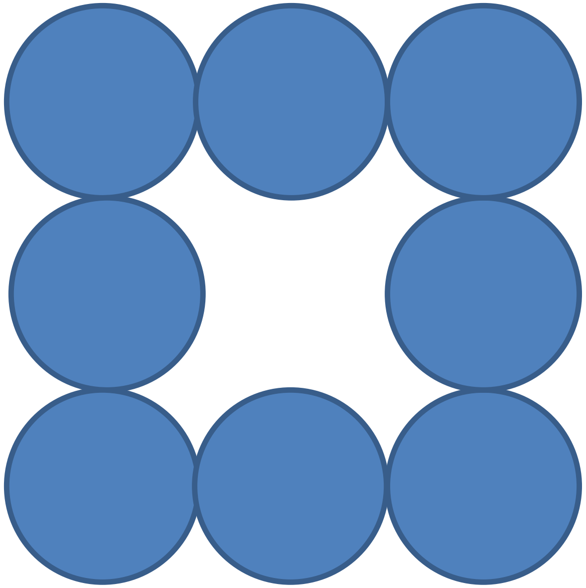
A good problem?

- Does this problem have multiple entries and exits?
- Does it have useful mathematics?
- Does it have links to the curriculum?

More Problems by Analogy I

- Using the same techniques:

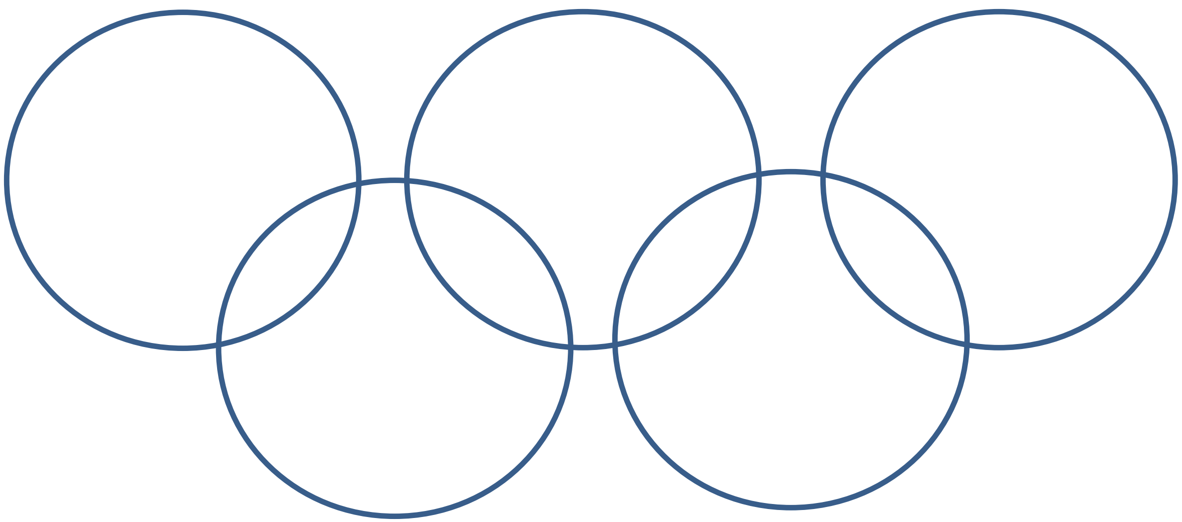
  8 Circles! (and extensions to other polygons)



More Problems by Analogy II

- This one looks the same and fundamentally is, but it has one or two little wrinkles that make it still a problem: Olympic Rings.

Here we use the weights 1, 2, 3, 4, 5, 6, 7, 8, 9.



Where to from here? Infinity?

- 6 circles can be an end in itself – can finish at getting 4 thingies or a proof that there are only 4 thingies
- Nice sets can be a place to stop – discover some patterns or discover a lot of patterns or see the general pattern or prove the general pattern
- Polygonal arrays of circles
- Olympic rings and all sorts of other shapes

Musings

- The way I've stated this problem is poor. There needs to be a better context, ***your*** ***context***

    for example a combination lock

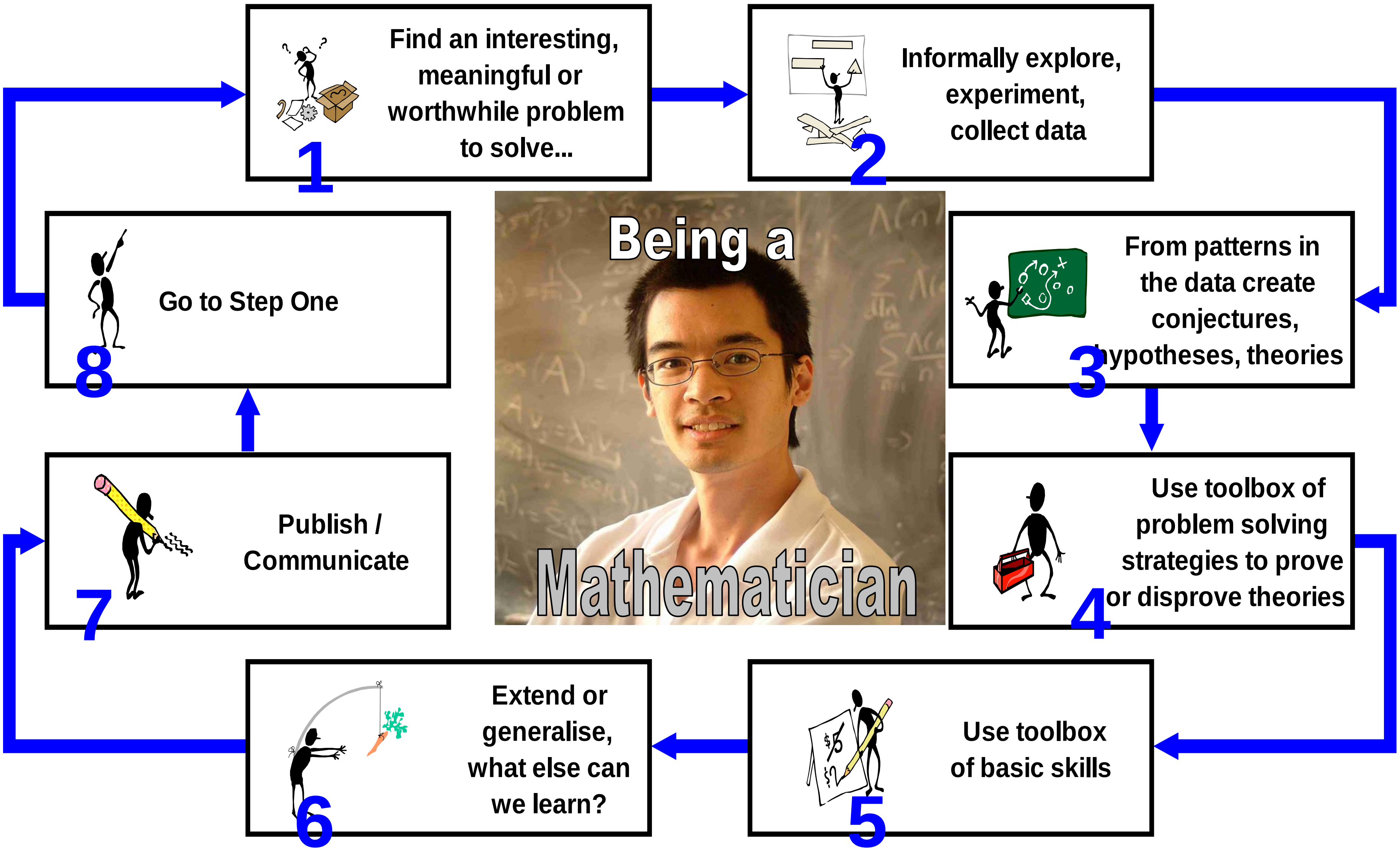
- Try to use a context that will interest your students and get them involved

    they'll remember the problem/method longer

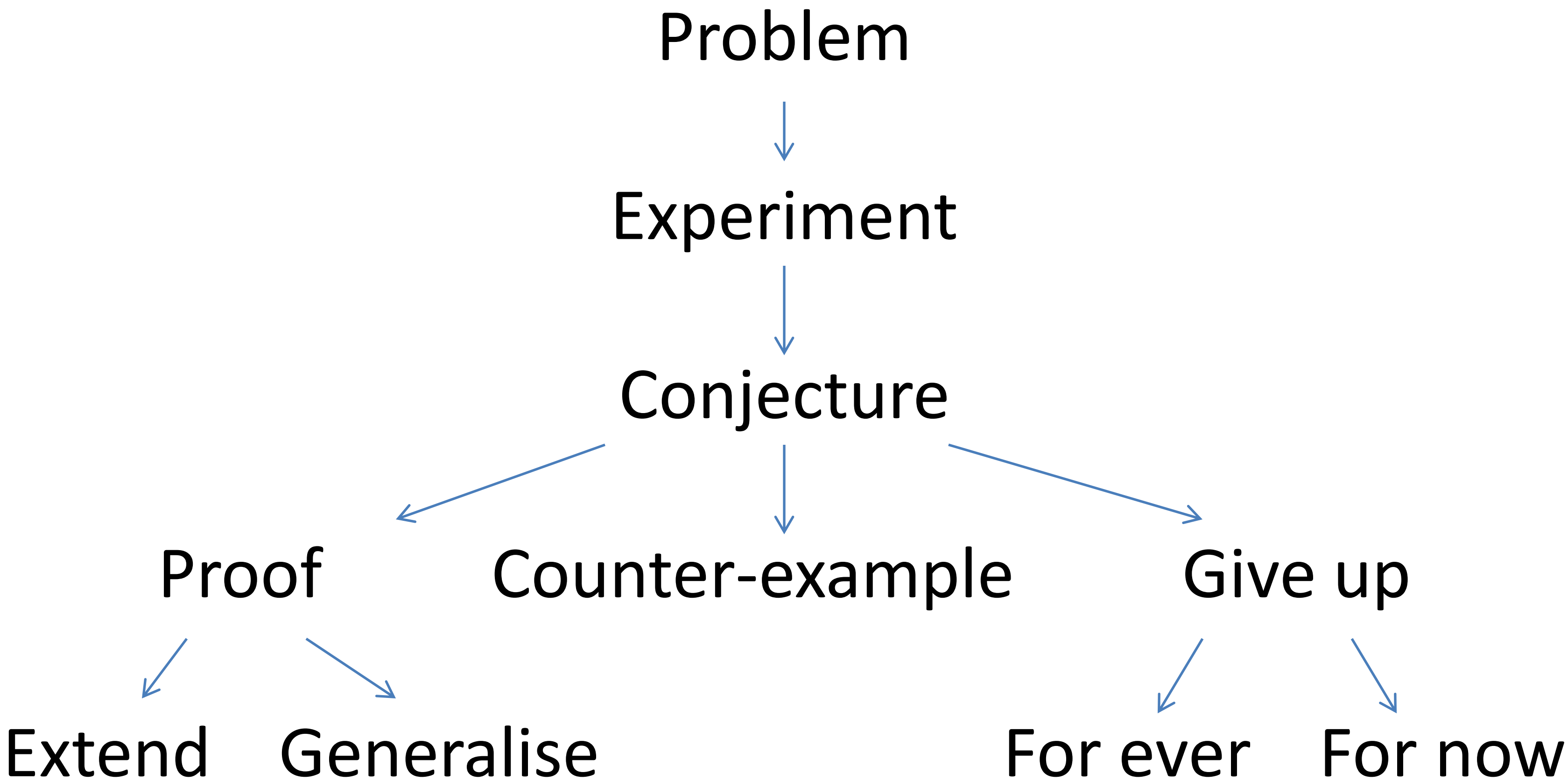
- So don't necessarily do it my way!

Use?

- What grades/years might you use this problem with?
- Would you do different things for the weaker students, the middle ability students and the brighter students?
- What would be your aims by doing this?
- How would you change the way you present this problem?


Find an interesting, meaningful or worthwhile problem to solve…
1
Informally explore, experiment, collect data
2
From patterns in the data create conjectures, hypotheses, theories
3
Use toolbox of problem solving strategies to prove or disprove theories
4
Use toolbox of basic skills
5
Extend or generalise, what else can we learn?
6
Publish / Communicate
7
Go to Step One
8
Being a Mathematician

How a mathematician works



Some useful web sites

- http://nrich.maths.org/frontpage
- http://www.maths300.esa.edu.au/
- http://www.nzmaths.co.nz/

References

- Holton, D. & Lovitt, C. (2013). *Lighting Mathematical Fires 2*. Curriculum Press. Melbourne, Australia: ESA.
- Holton, D. (2012). *Problem Solving: The creative process*. Mathematical Association UK.
- Holton, D. (2013). *More Problem Solving: The creative process*. Mathematical Association UK.