

St.Kilda Road and Swanston Street Maths Trail

THE FLORAL CLOCK



Figure 6.1. The Floral Clock.

Figure 6.1 shows the floral clock in the Queen Victoria Gardens in St. Kilda Road. The floral clock has a radius of four metres, with an inner circle of radius 2.5 metres (Figures 6.2 and 6.3). The lengths of the minute hand and the hour hand are 2.1 metres and 1.7 metres respectively.

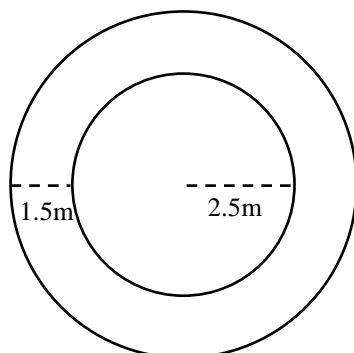


Figure 6.2. Dimensions of the inner and outer circle of the Floral Clock.

Figure 6.3. The clock in preparation for a new planting.

Flowers are planted to form geometric designs in the inner circle. Depending on the design, approximately 4000 plants are needed for the inner design and 3000–3500 for the outer section. The designs are frequently based on polygons. Figures 6.4 and 6.5 show two designs based on the octagon which have been reproduced with the permission of the landscape designer, Sam Davis.



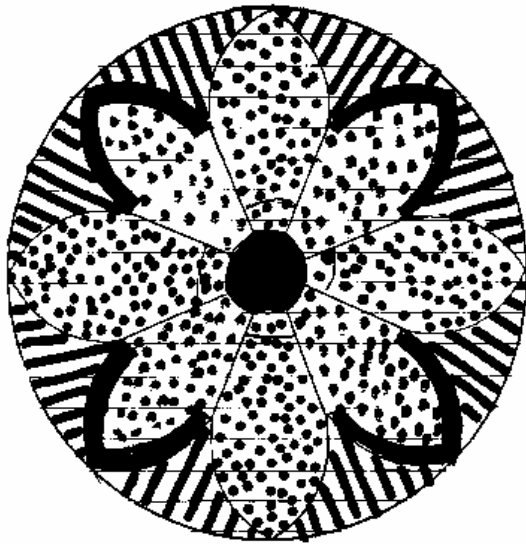


Figure 6.4. Flower design.

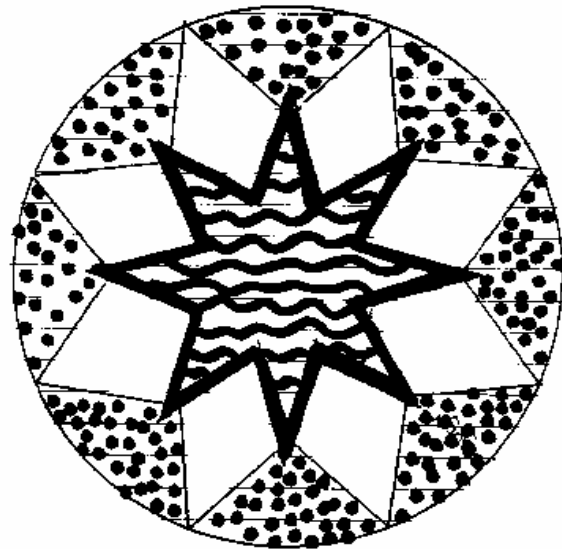


Figure 6.5. Double star design.

Figures 6.6 and 6.7 show the geometric construction of these designs based on an octagon.

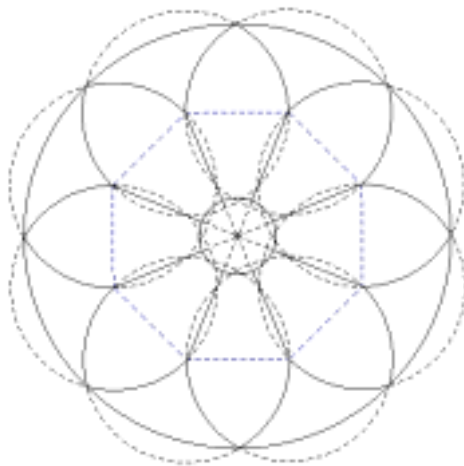


Figure 6.6. Geometric construction of the flower design.

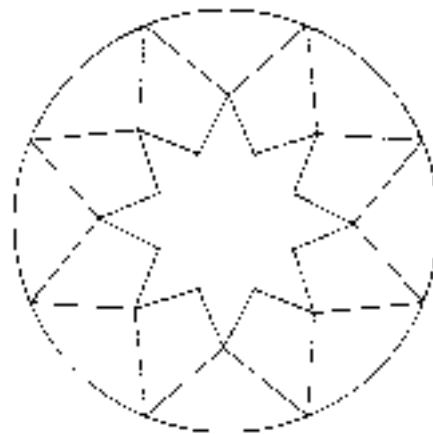


Figure 6.7. Geometric construction of the double star design.

PEDESTRIAN BRIDGE AT SOUTHGATE

The pedestrian bridge over the Yarra River at Southgate (Figure 6.8) was designed by the Melbourne architects, Cocks and Carmichael. The curve of the arch is a parabola.



Figure 6.8. The footbridge from Princes Bridge.

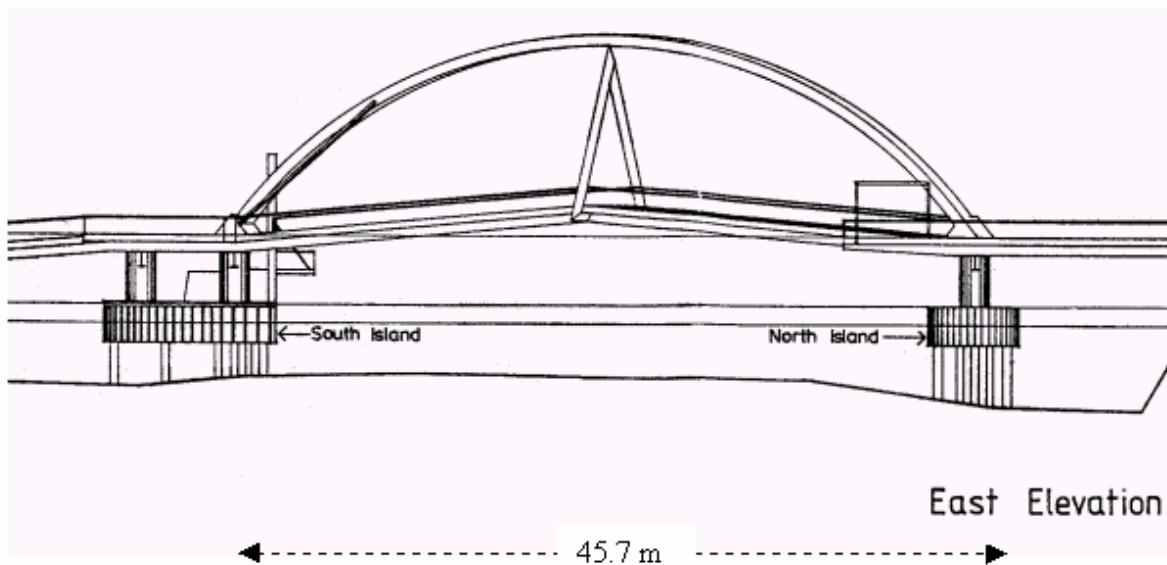


Figure 6.9. East elevation of the footbridge (Architects' drawing).

FEDERATION SQUARE

Federation Square (see Figure 6.10) was designed by architects Peter Davidson and Don Bates of *Lab architecture studio* in association with *Bates Smart Architects*. Their winning design was chosen by the judges from a total of 177 entries in the Federation Square Design Competition. The complex of buildings features over 22, 000 tiles, constructed from German zinc, Western Australian sandstone and local glass.

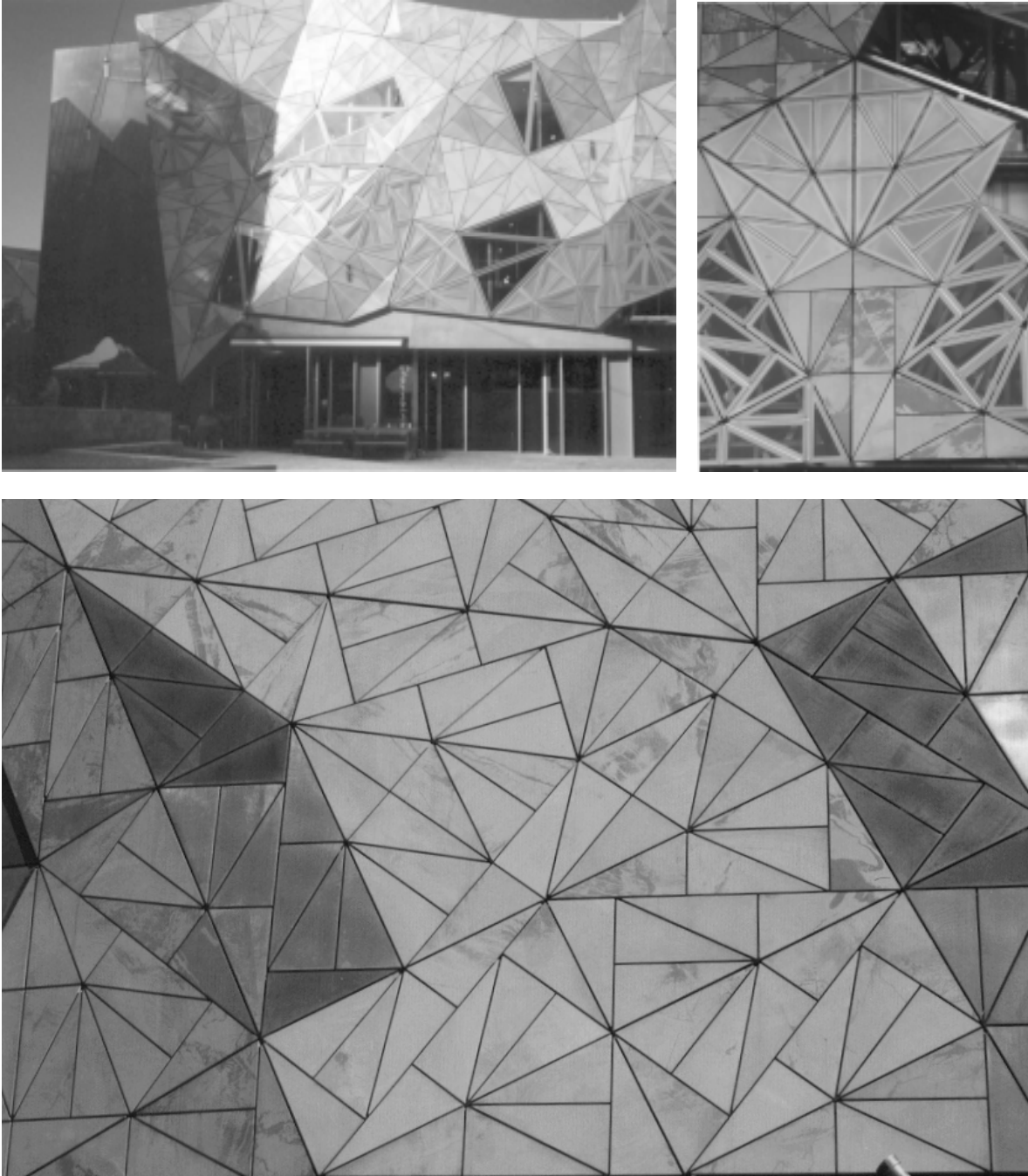


Figure 6.10. Federation Square.

PINWHEEL TILING

The design is based on a single triangle – a right-angled triangle – with sides in the ratio $1 : 2 : \sqrt{5}$ (see Figure 6.11), described by Charles Radin of the University of Texas in 1994 in the journal, *Annals of Mathematics*. Radin attributes the tiling to John H. Conway. Besides the right-angle, the other two angles in the triangle are $\tan^{-1}(2)$ and $\tan^{-1}(1/2)$, that is, the angles whose tangents are 2 and $1/2$. These angles are irrational, so rotating the tile will never bring it back exactly to its initial position. Therefore there are an infinite number of possible rotations of the tiles, giving rise to the name *pinwheel tiling*. Figure 6.12 shows the triangle rotated through the angle $\tan^{-1}(1/2)$ about the point P .

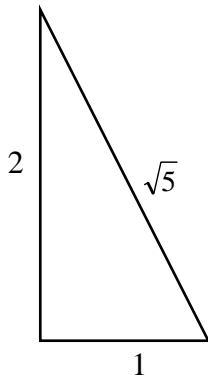


Figure 6.11. The $1 : 2 : \sqrt{5}$ triangle.

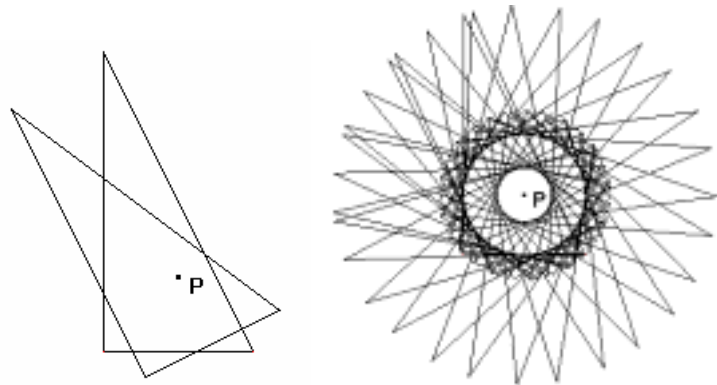


Figure 6.12. Rotation of the triangle about point P .

PERIODIC AND NONPERIODIC TILINGS

If a tiling is **periodic**, it is possible to make a copy of part of the tiling on transparent film and slide the copy (without rotation) to exactly match up with another region of the tiling. The tiling of octagons and squares in Figure 6.13 is a periodic tiling.

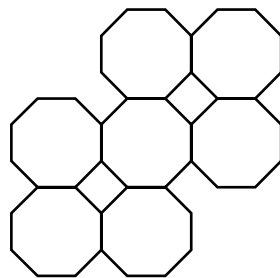
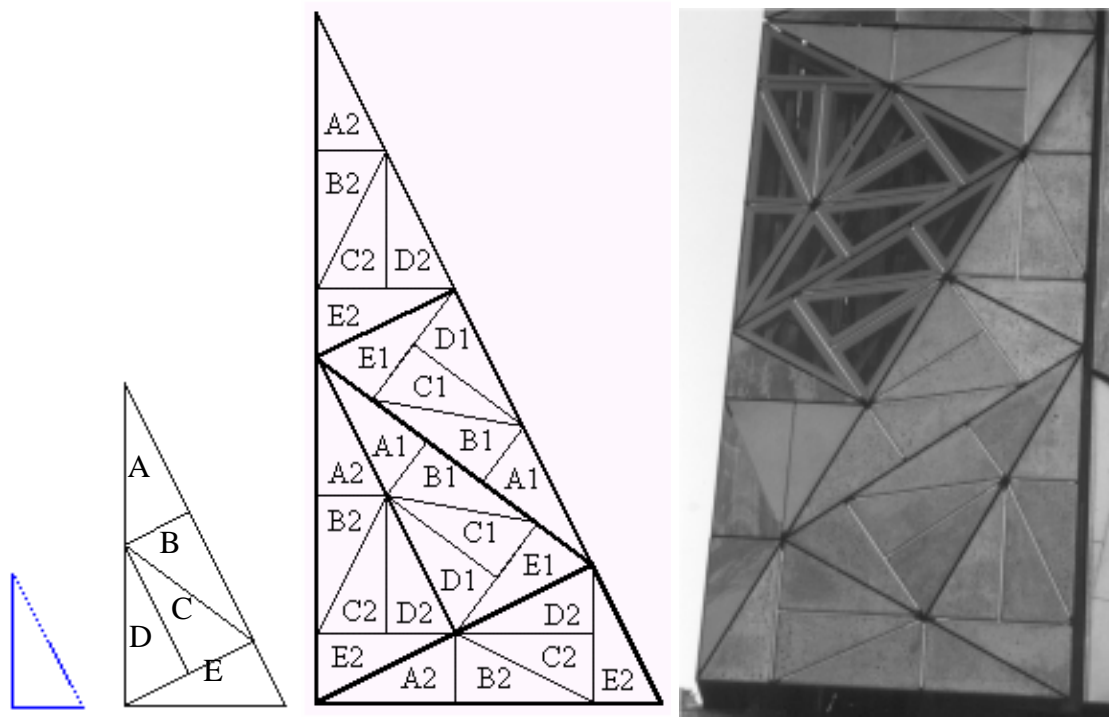


Figure 6.13. Periodic tiling of octagons and squares.

A **nonperiodic** tessellation may be likened to an irrational number—such as the square root of 2 or 3—where the decimal places go on forever without ever repeating exactly the same sequence. In the case of the pinwheel tiling, by rotation, reflection and translation, five tiles form a larger, similar right-angled triangle tile whose sides are $\sqrt{5}$ times the lengths of the sides of the original triangle (see Figure 6.14). When this process is repeated, a nonperiodic, or nonrepeating, tiling is formed. The five Level 0 tiles (see Figure 6.14a) which make up the larger, similar Level 1 triangle in Figure 6.14b are identified as A, B, C, D, and E according to their position in the larger triangle. At the next level, Level 2 (Figure 6.14c), for tile position A, for example, A1 and A2 represent reflections of each other. The tilings shown in Figures 6.14 to 6.16 have been constructed in MicroWorlds Pro using the Logo programming language.



(a) Level 0 (b) Level 1 (c) Level 2

Figure 6.14. Successive levels of the pinwheel tiling.

This type of tiling is known as a substitution tiling, since at each successive level, tiles from the previous level are substituted into a larger similar shape. At each successive level, a greater number of rotations of the triangles occurs, as seen in Figures 6.15 – 6.16.

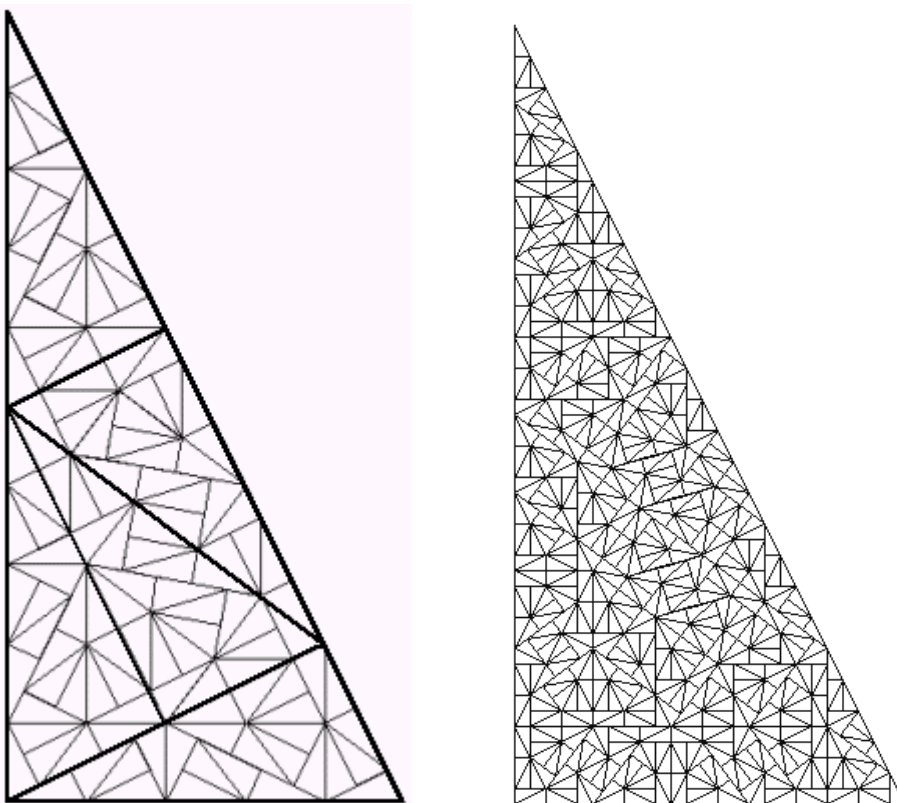


Figure 6.15. Levels 3 and 4 of the pinwheel tiling.

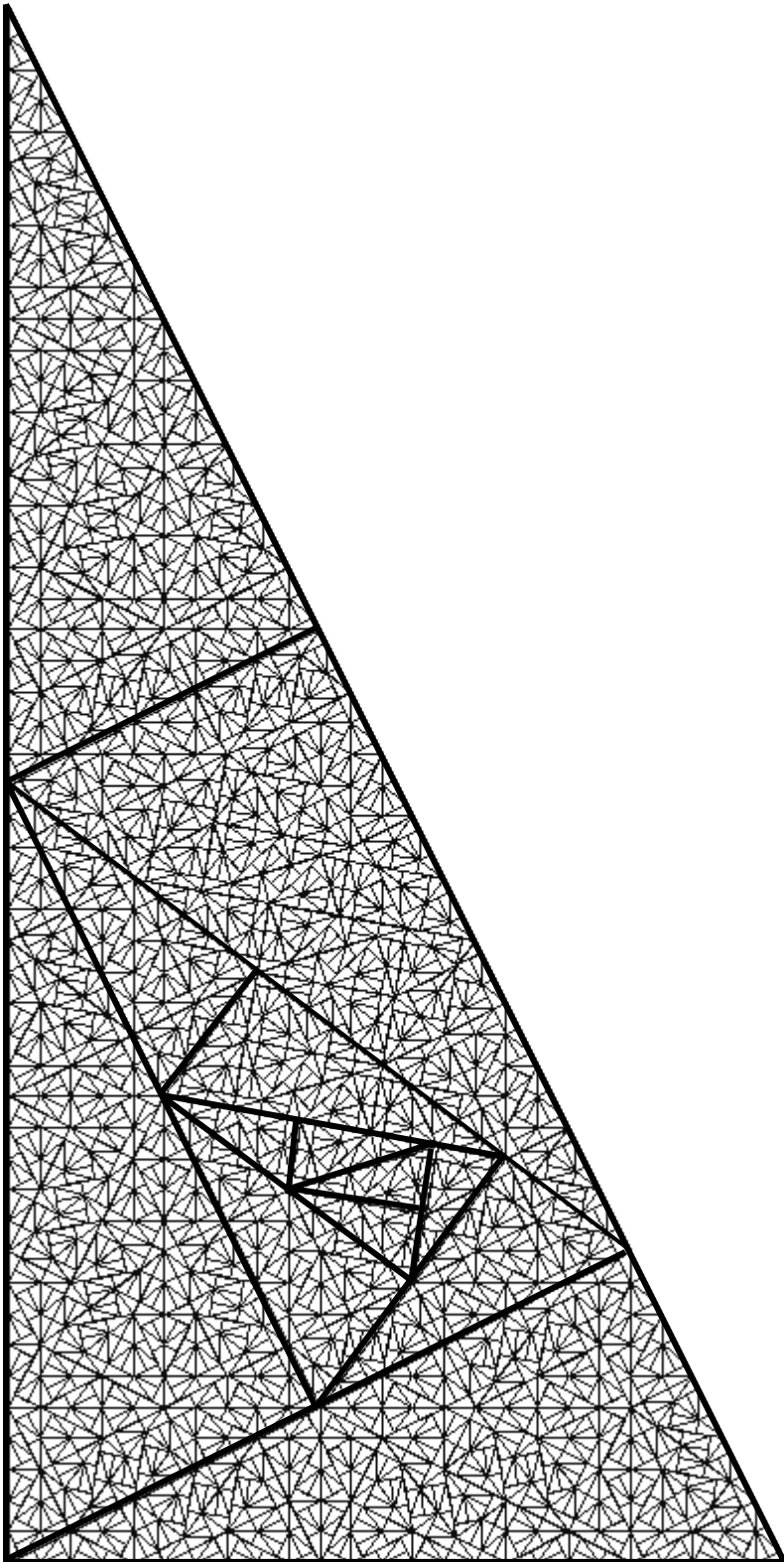


Figure 6.16. Level 5 of the pinwheel tiling, showing Levels 3 and 4 for the triangle in position C.

See the following website for further information about the pinwheel tiling:
<http://www.ma.utexas.edu/users/radin/federation/index.html>

ST. PAUL'S CATHEDRAL

The arches and windows of St. Paul's Cathedral have designs constructed from circles and polygons (Figure 6.17). These geometric designs originated in about the 12th century in the Gothic architecture of cathedrals and churches in Europe. The equilateral triangle and the square gave rise to many designs based on three and four circles or arcs, symbolising the Trinity and the four Gospels. In later Gothic architecture, patterns were often based on five, six, seven or eight circles.



Figure 6.17. St. Paul's Cathedral: Arches and circles.

GOTHIC ARCHES

The *equilateral arch* is constructed on the equilateral triangle ABC by drawing two circles with their centres at A and B , intersecting at C (see Figure 6.18).

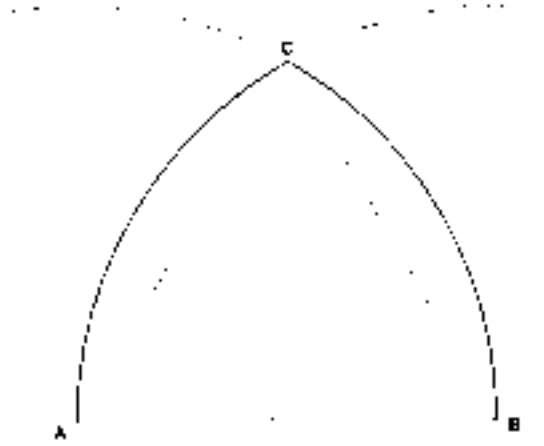


Figure 6.18. Construction of an equilateral arch.

The *trefoil arch* (Figure 6.19) is constructed from an equilateral triangle with the centres of three circles at the midpoints of the sides (L, M, N).

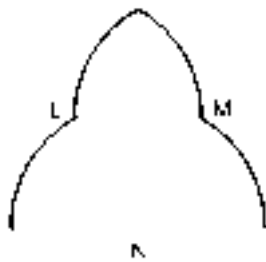


Figure 6.19. Trefoil arch.

Trefoils, designs based on the equilateral triangle and circles, may be constructed from intersecting circles as in Figure 6.20 (a) and (b) or circles which touch each other tangentially as in (c).

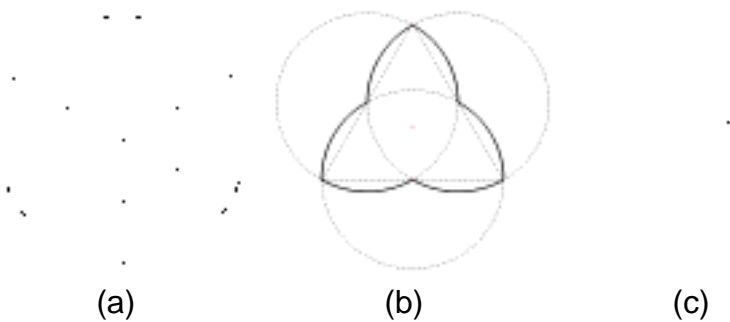


Figure 6.20. Trefoils.

QUATREFOILS

Quatrefoils, designs based on four circles, may be constructed from *intersecting circles* (Figure 6.21) or from *tangent circles* (Figure 6.22).

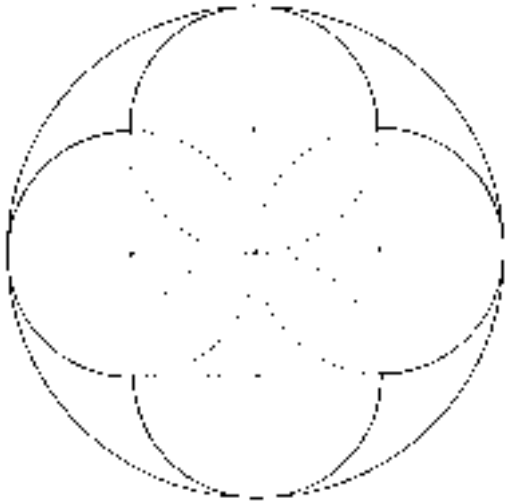


Figure 6.21. Intersecting circles.

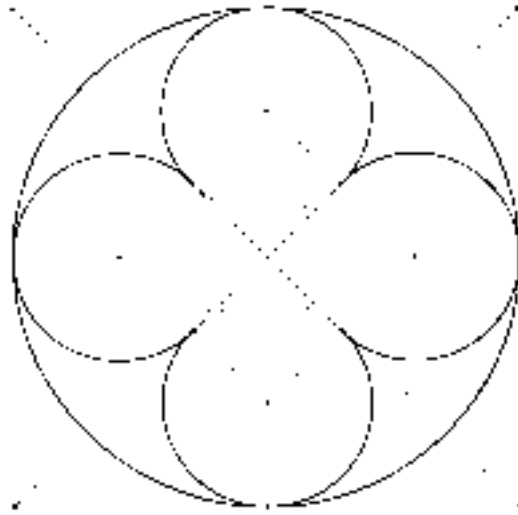


Figure 6.22. Tangent circles.

OTHER PATTERNS BASED ON TANGENT CIRCLES

The tangent circle designs in Figure 6.23 are all based on regular polygons. Geometric constructions based on regular pentagons, hexagons and octagons would have been known to medieval architects.

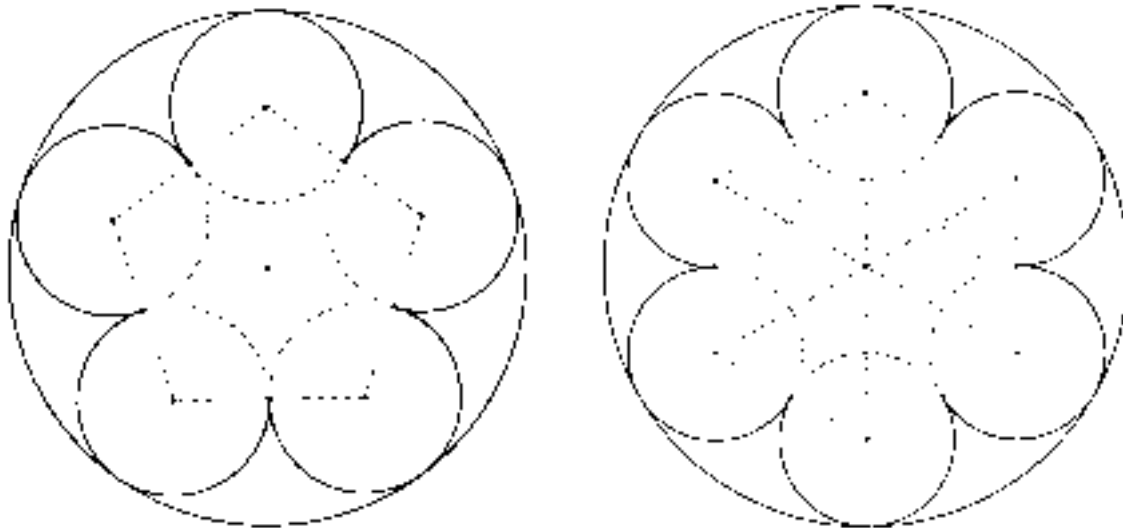


Figure 6.23. Tangent circles around polygons.

MELBOURNE TOWN HALL

Some of the doors and windows of the Melbourne Town Hall are semicircular, with an inscribed circle in the semicircle (Figure 6.24). The semicircle and its base are tangential to the inscribed circle.

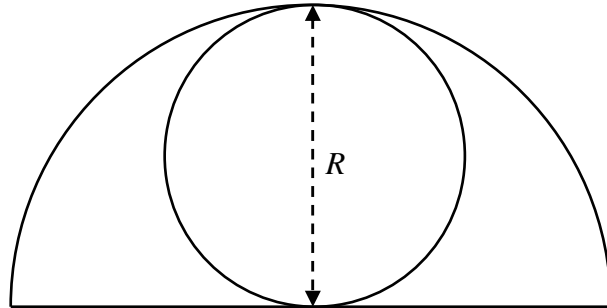


Figure 6.24. Semicircular arch with inscribed circle at Melbourne Town Hall.

If the radius of the semicircle is R , the radius of the inscribed circle is $R/2$.

Length of semicircular arc = Circumference of inscribed circle = πR

It can be shown that the area of the inscribed circle is half the area of the semicircle (see Mathematical Activities). Smaller tangent circles can be included to produce the design shown in Figure 6.25. If the radius of the semicircle is R , the radius of each of the two small tangent circles is $R/4$ (see Mathematical Activities).

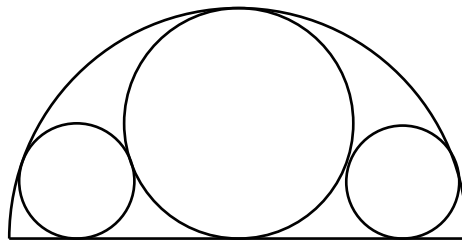


Figure 6.25. Semicircle with three tangent circles.

An interesting geometric shape based on semicircles is the *Shoemaker's Knife* or *Arbelos* (Figure 6.26), which was studied by Archimedes (287–212 BC). The arbelos is formed by three semicircles, with the two inner semicircles tangent to each other and to the outer semicircle. It can be shown that the length of the outer semicircle arc is equal to the sum of the lengths of the two smaller semicircle arcs (see Mathematical Activities).

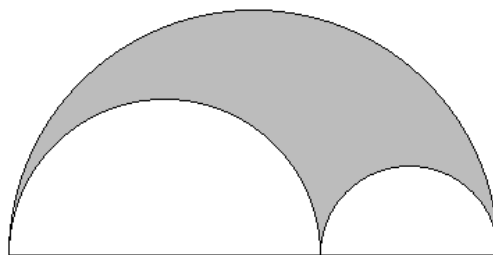


Figure 6.26. The Shoemaker's Knife or Arbelos.

ARCHITECTURAL FRAGMENT

Outside the State Library is the bluestone sculpture *Architectural Fragment* (Figure 6.27) by Victorian sculptor, Petrus Spronk. Inspired by the facade of the State Library, the sculpture was commissioned by the City of Melbourne in 1992 as the first of a number of street sculptures in Swanston Street. The basic triangular pyramid shape of the sculpture is constructed from steel, with the Port Fairy bluestone slabs bolted and cemented into place. The sculpture was constructed at the quarry and stone-cutting plant (Figure 6.28) then transported to Swanston Street, where it was set into the pavement.



Figure 6.27. Architectural Fragment.

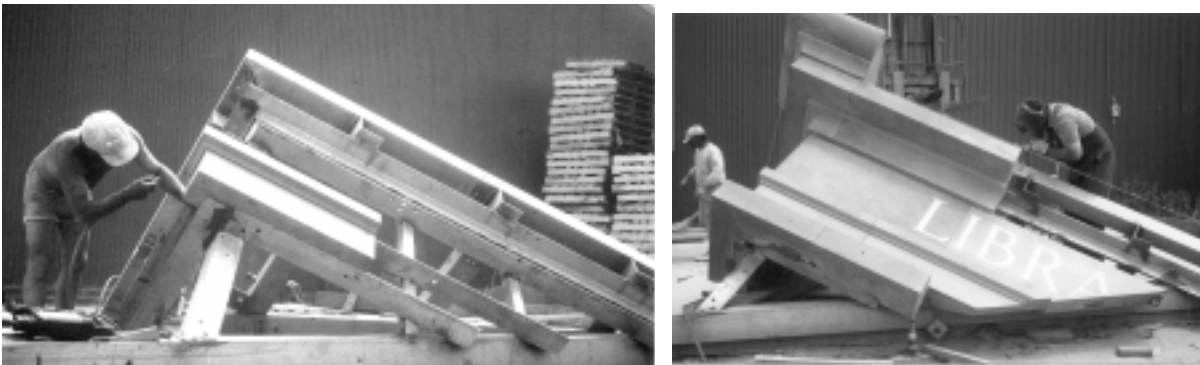


Figure 6.28. Construction of Architectural Fragment.

Petrus describes his concept of Architectural Fragment:

“When one day I was made aware of a brief for a public work for Swanston Walk, I knew exactly what it was I should propose. I proposed a fragment in stone. It was accepted. Initially the sand works and fragment’s idea came from my fascination with the fragments of sculpture I found lying about everywhere on the island of Samos, some dating back 6000 years. This is the island of Pythagoras. Walking through these areas I was awestruck at both the beauty and the age of the architectural fragments I came upon. A piece of a building discarded in the landscape. That was many years ago.

While thinking about a work for Swanston Walk, I realised it had to be visually strong. So I chose the strongest form we have, the triangle. It is plain and strong, both visually and physically. I based this on Pythagoras’ triangle using the simple formula of 3:4:5, each unit represented by one metre. I isolated the work from as many buildings as I could and consequently placed it where it now is. I had the choice of the street. But since the work needed space this was the only place, lest it be overwhelmed by other buildings close by.

I made a triangle from rope, with three loops in it so that the space between the loops was 3:4:5, in metres. I took this length of rope and two screwdrivers to Swanston Street and placed the 5 metres along the footpath and stuck the two loops into the footpath with the screwdrivers. I got my friend to hold up the triangle by the third loop and asked her to keep adjusting it up and down. I walked about, looked at it from close by, from a distance, from across the road. I climbed the stairs to the third floor of the building across the road to see it from that vantage point and finally decided on the correct angle, purely by eye, by my feeling for the correct and strongest result.

I then measured the distance between the point my friend was holding up and the footpath. This gave me the second triangle. The large triangle on the back was a result of the first two. Initially this last triangle could have provided me with a design problem, because it represents the roof of the fragment of the building depicted. I overcame this by creating it as a continuation of the footpath pattern, thus paying homage to the way bluestone has always been used.”

When the sculpture was placed in position in Swanston Street, it was set deeper into the footpath than originally intended, with the result that the dimensions of the triangles observed in the sculpture vary slightly from those shown in the drawings in Figures 6.29 – 6.30.

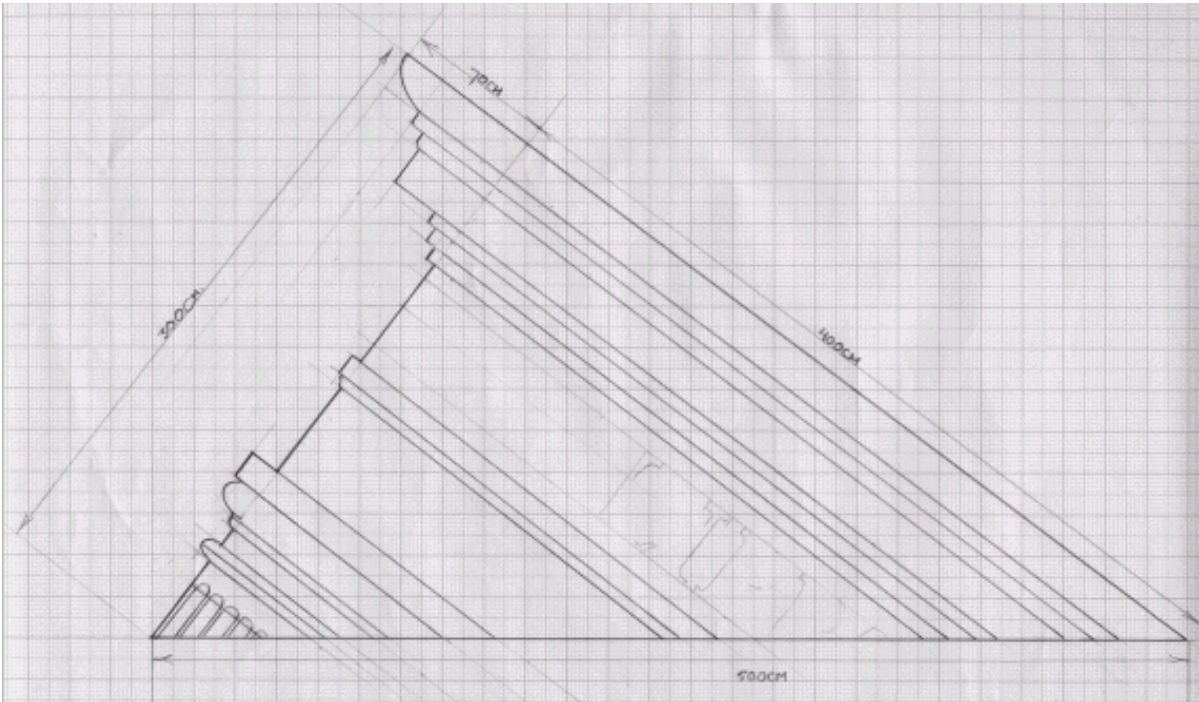


Figure 6.29. Side elevation of Architectural Fragment showing the 3:4:5 triangle.

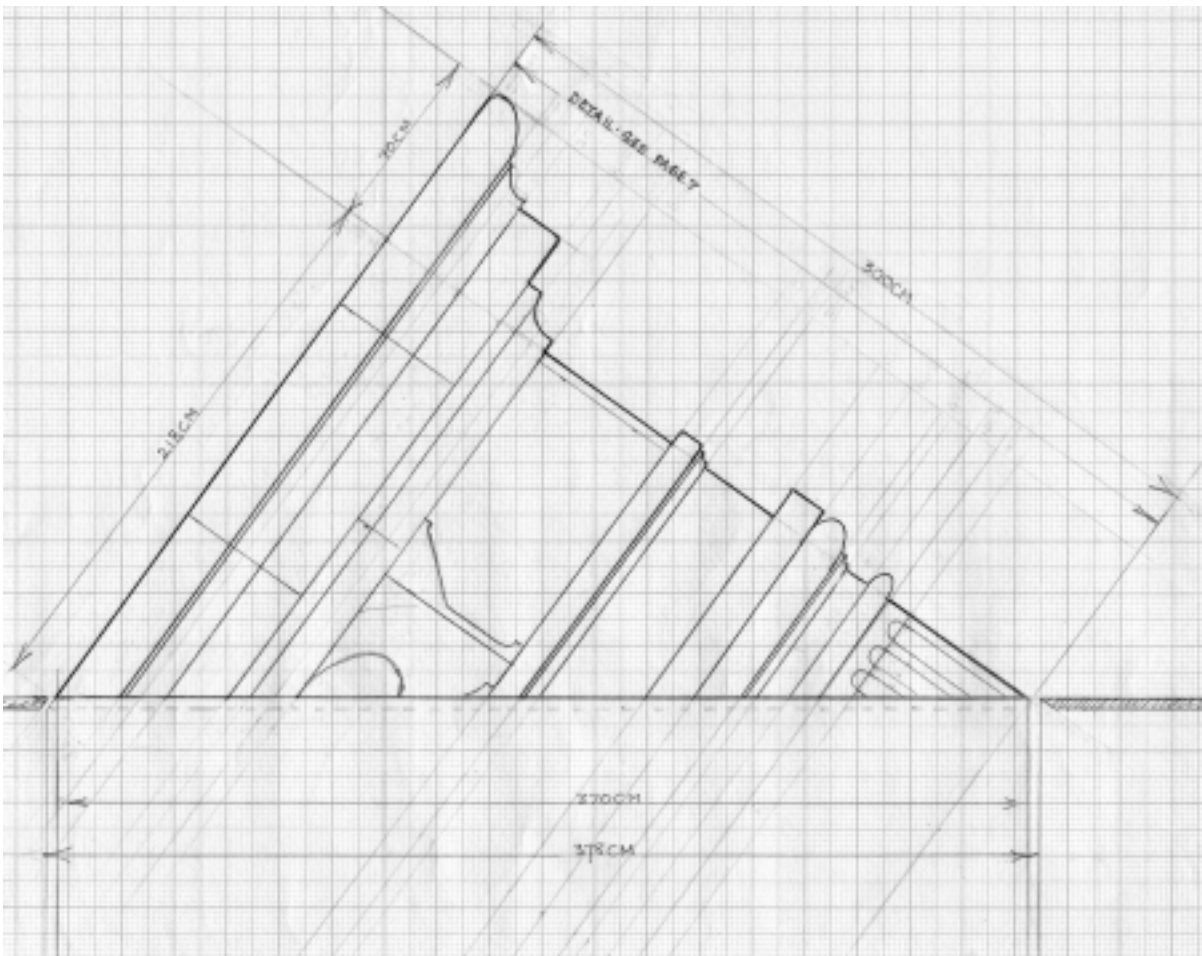


Figure 6.30. Side elevation of Architectural Fragment with 218 cm, 300 cm, 370 cm triangle.

MELBOURNE CENTRAL CONE

Constructed of steel and glass, the cone at Melbourne Central (Figure 6.31) surrounds the historic red-brick shot tower. It was designed by the Japanese architect, Kisho Kurokawa, in conjunction with Melbourne architects, Hassell Pty. Ltd. Kurokawa has produced highly acclaimed works in over 20 countries.

In the Shot Tower, lead ingots were heated in a furnace to melt them, then the molten lead was dropped from the top of the tower into a one metre deep trough of water at the bottom. As the molten lead fell, it formed into spherical balls that solidified as they landed in the water.



Figure 6.31. The Shot Tower inside the cone.

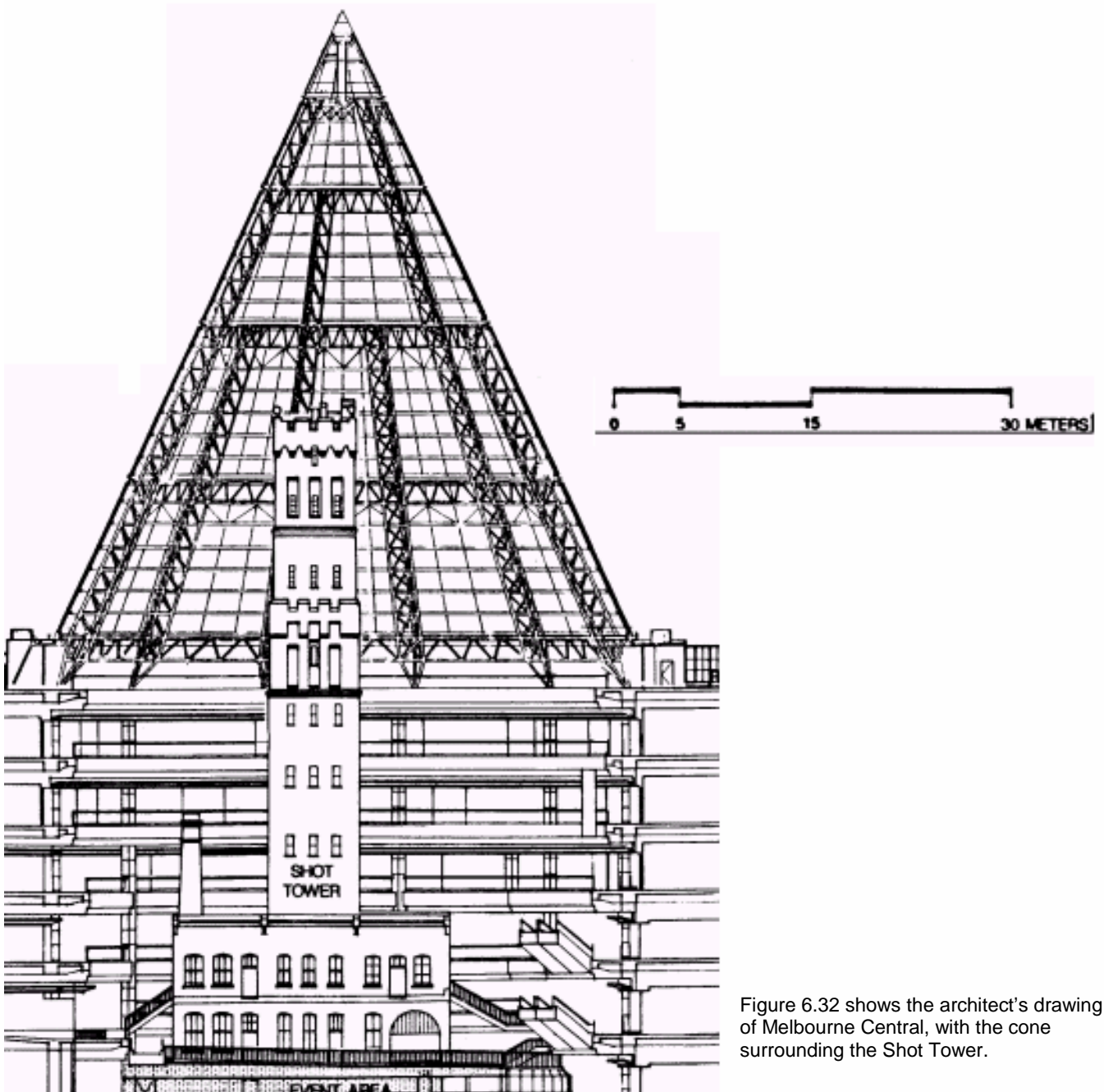


Figure 6.32 shows the architect's drawing of Melbourne Central, with the cone surrounding the Shot Tower.

STOREY HALL, RMIT

Storey Hall, RMIT, was designed by Melbourne Architects, Ashton Raggatt McDougall. The colours of Storey Hall, green and purple, reflect its former association with the Suffragette Movement and the Irish Hibernian Society in the early 20th century. The most striking feature of the architecture is the pattern of rhombuses creeping up the facade of the building and over the walls and ceiling inside (Figure 6.33).

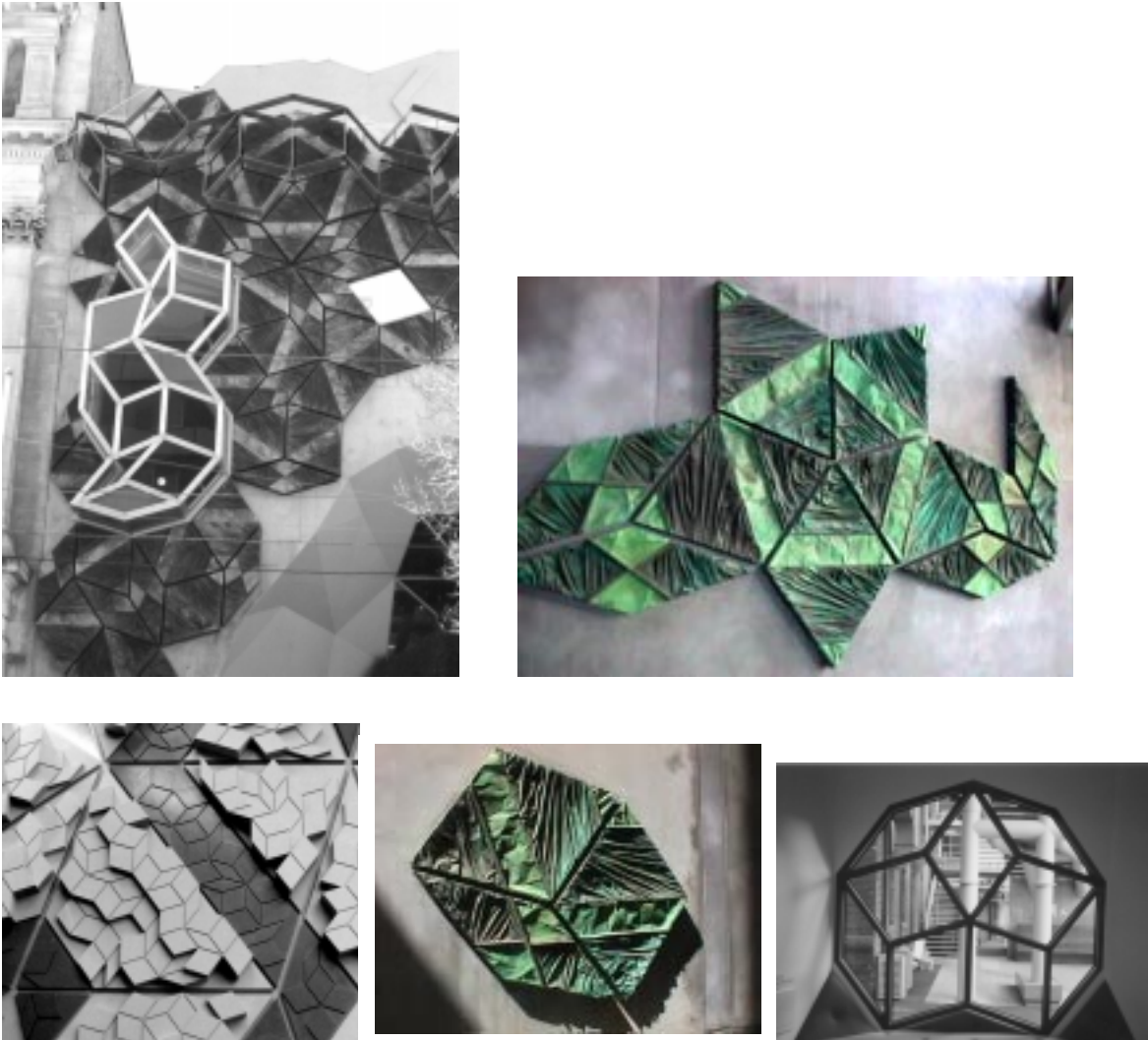


Figure 6.33. Penrose rhombuses at Storey Hall.

The architects, describing the public reaction to the facade of the Storey Hall Annex, write:

“... we were told that the street seemed uncontrollably joyful. People smiled, people laughed, people stopped mouth open, people talked to strangers, people waved their arms about, people walked up and down, people came back again, people pointed, people fell off their bikes, people stopped in their cars, people almost got run over by trams, people actually looked at a building, people went over and touched it, people had an opinion.”

RMIT Storey Hall. Transition, Issue 51, 1996: Faculty of Design and Construction, RMIT, p.9.

SIR ROGER PENROSE

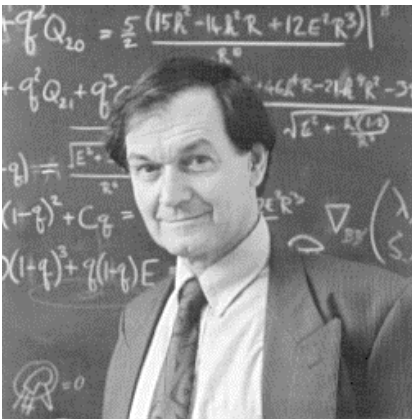


Figure 6.34. Sir Roger Penrose.

Sir Roger Penrose (Figure 6.34) is a Professor of Mathematics at Oxford University. His main work is in relativity and quantum mechanics. Penrose was knighted and was joint winner with Stephen Hawking of the 1988 Wolf Prize for Physics for his services to our understanding of the universe. Like his geneticist father, he has always been interested in mathematics as a recreation and one of his interests has been in exploring tiling patterns which are nonperiodic, that is, tiles which do not form repeating patterns (Federation Square is also based on a nonperiodic tiling). In 2000 Penrose was invited to Melbourne to speak at the *Maths 2000 Festival* and he saw Storey Hall for the first time.

In 1973 Penrose found a set of six tiles which tessellate nonperiodically. He noted that he may have been influenced by his memories of seeing tiling patterns based on pentagons in his father's copy of Kepler's *Harmonice mundi* (1619). In 1974 Penrose reduced the set of six tiles to just two - a 'fat' rhombus with angles of 72° and 108° and a 'thin' rhombus with angles of 36° and 144° (Figure 6.35).

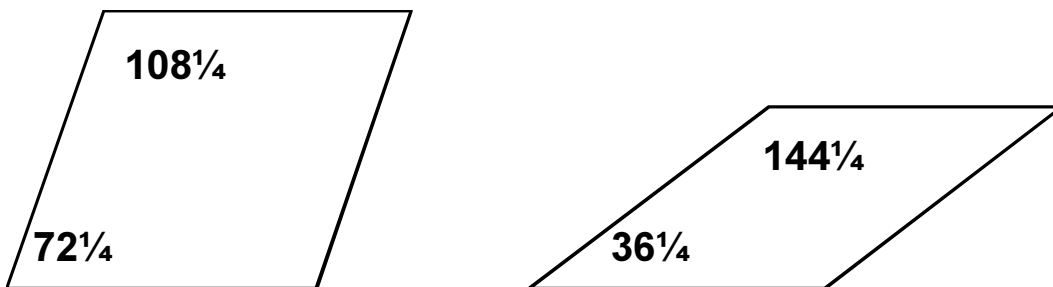


Figure 6.35. Fat and thin rhombuses.

Although the rhombuses can tessellate to form a repeating pattern, as shown in Figure 6.36, they can also tessellate nonperiodically.

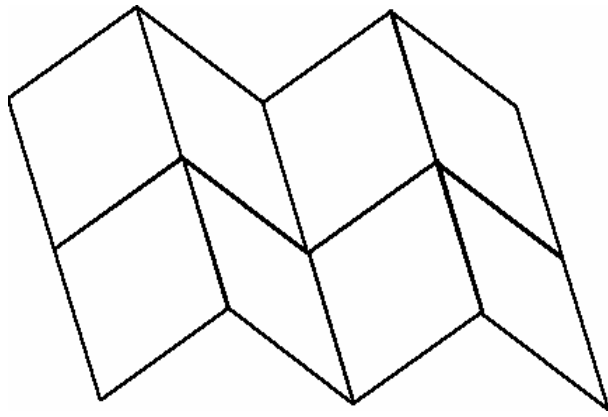


Figure 6.36. Repeating pattern formed by fat and thin rhombuses.

The bands shown on the rhombuses in Figure 6.37, which serve the same function as the bumps and dents on the pieces of a jigsaw puzzle, allow the fat and thin rhombuses to fit together in only one way, forcing a nonperiodic tiling. It is this Penrose nonperiodic tiling which forms the basis of the architecture of Storey Hall. Inside the Storey Hall auditorium, emerald green bands on the white rhombuses form pentagonal patterns which are repeated in fractal-like fashion over the walls and ceiling.

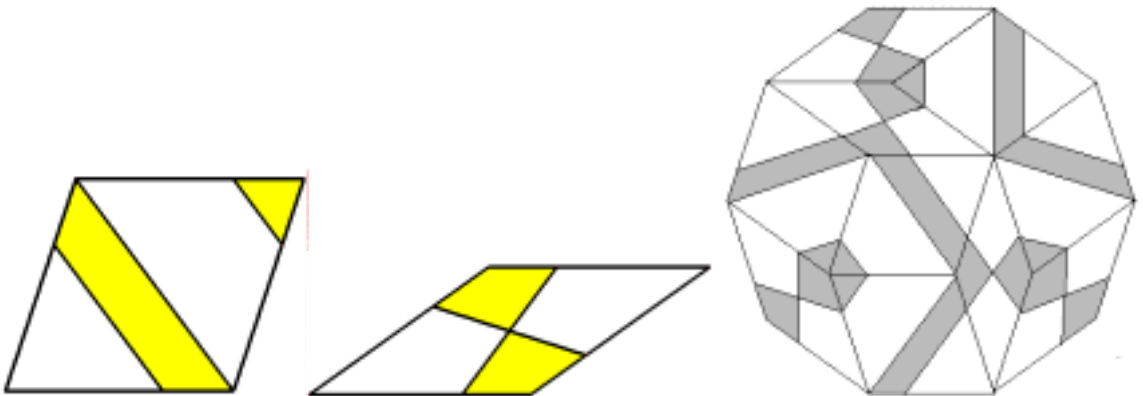


Figure 6.37. Pattern of bands which forces a nonperiodic tiling.

The rhombuses tessellate to form decagons (see Figure 6.38), creating an impression of 3-dimensional cubes. Overlapping decagons form the nonperiodic tiling shown in Figure 6.39.



Figure 6.38. Decagons formed by fat and thin rhombuses.

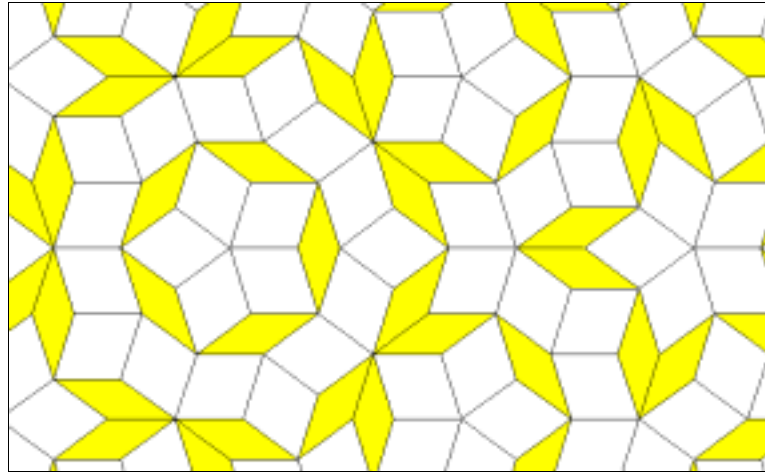


Figure 6.39. Nonperiodic tiling of fat and thin rhombuses.

If the long diagonal of the fat rhombus ($72^\circ/108^\circ$ rhombus) is divided in the golden ratio, the rhombus can be divided into four isosceles triangles (see Figure 6.40a). By combining the triangles as shown in Figure 6.40b, Penrose produced two further shapes which can be forced to tessellate nonperiodically. John Conway named the two shapes *kites* and *darts*.

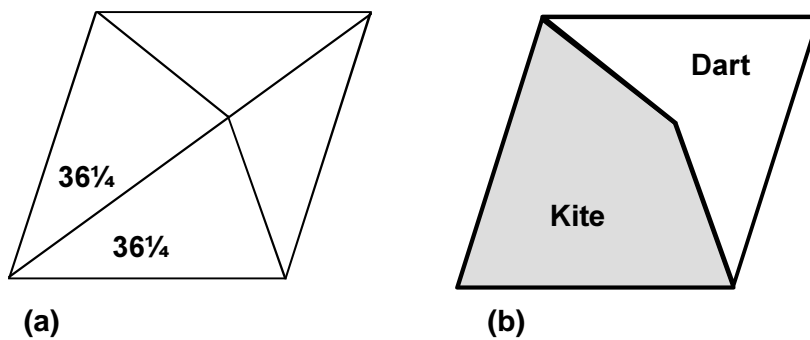


Figure 6.40. Dividing the long diagonal of the fat rhombus in the golden ratio.

The kites and darts form another nonperiodic tessellation, as shown in Figure 6.41.

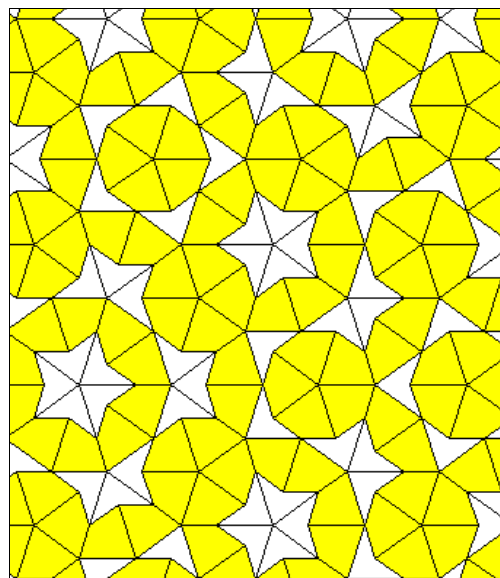


Figure 6.41. Nonperiodic tessellation of kites and darts.

Of course, the kite and dart can be combined to form the original fat rhombus that can then tile periodically, as shown in Figure 6.42. In order to produce a forced nonperiodic tiling, Conway proposed the drawing of arcs on the kites and darts, as shown in Figure 6.43.

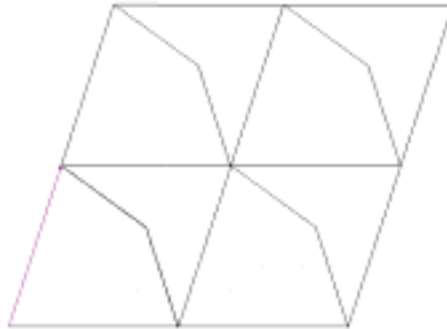


Figure 6.42. Periodic tiling formed from kites and darts.

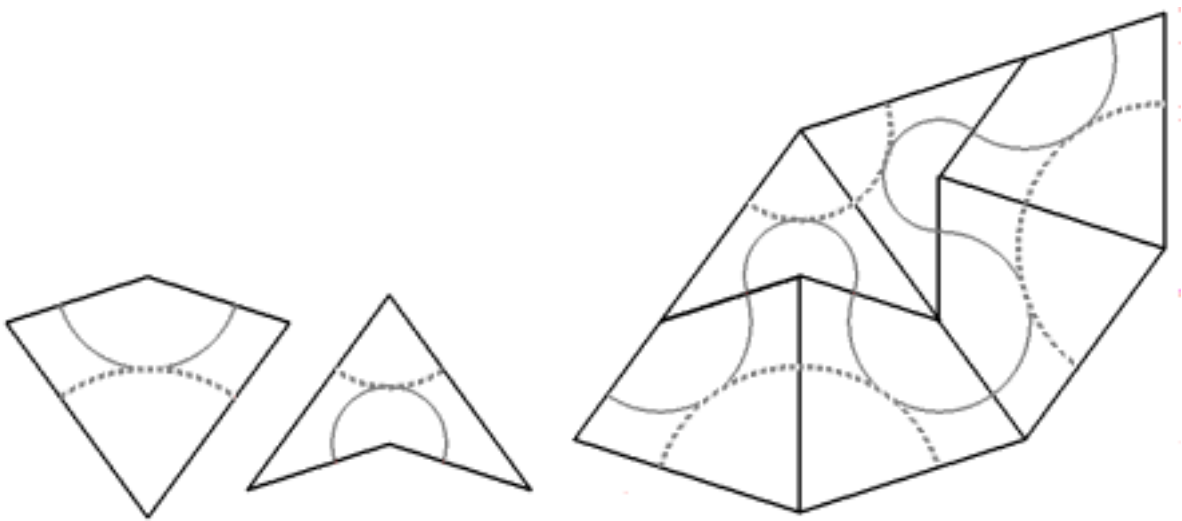


Figure 6.43. Using circular arcs to determine tilings with forced nonperiodicity.

Starting with kites and darts around a vertex, sometimes there is only one possibility—perhaps a dart or perhaps a kite—but sometimes there is a choice of adding either a kite or a dart. After adding a few more tiles, however, it may become impossible to add either tile. This means going back to the point where there appeared to be a choice and choosing the other tile. Penrose recognised that tilings with kites and darts could form the basis of a game so the shapes were patented.

Here are some Internet sites about Penrose and his tilings:

<http://www.geom.umn.edu/apps/quasitiler/start.html>

http://www.ntticc.or.jp/pub/ic_mag/ic025/html/109e.html

<http://www2.spsu.edu/math/tile/aperiodic/penrose/penrose1.htm>

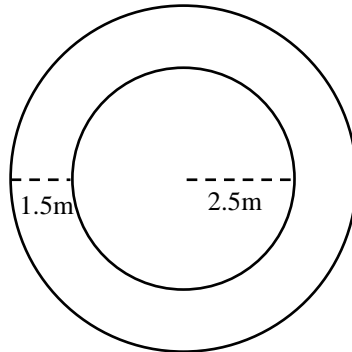
<http://www2.spsu.edu/math/tile/aperiodic/cartwheel/cartwheel1.htm>

<http://astronomy.swin.edu.au/~pbourke/texture/nonperiodic/>

Activities

THE FLORAL CLOCK

1. Calculate the areas of the inner circle and the outer section of the floral clock.



2. If the average number of plants required for the inner circle is 4000, how many plants per square metre does this represent?

3. Estimate how many of each of the four types of plants are needed for the flower and double star designs in Figures 6.4.

4. If the average number of plants required for the inner circle is 4000, how many square centimetres per plant does this represent?

5. Each planting of the clock requires a total of about 7000 plants. The cost of the plants is approximately \$35 per 144 plants. When purchasing the plants, the gardeners need to allow a loss of about 10% for seedlings which die. Calculate the approximate cost of planting the clock.

6. It takes four gardeners about four days to replant the clock. This includes pulling out the old design, replacing soil, designing and pegging out the new design and planting. If each gardener works 7.5 hours each day, how many hours does it take to replant the clock?

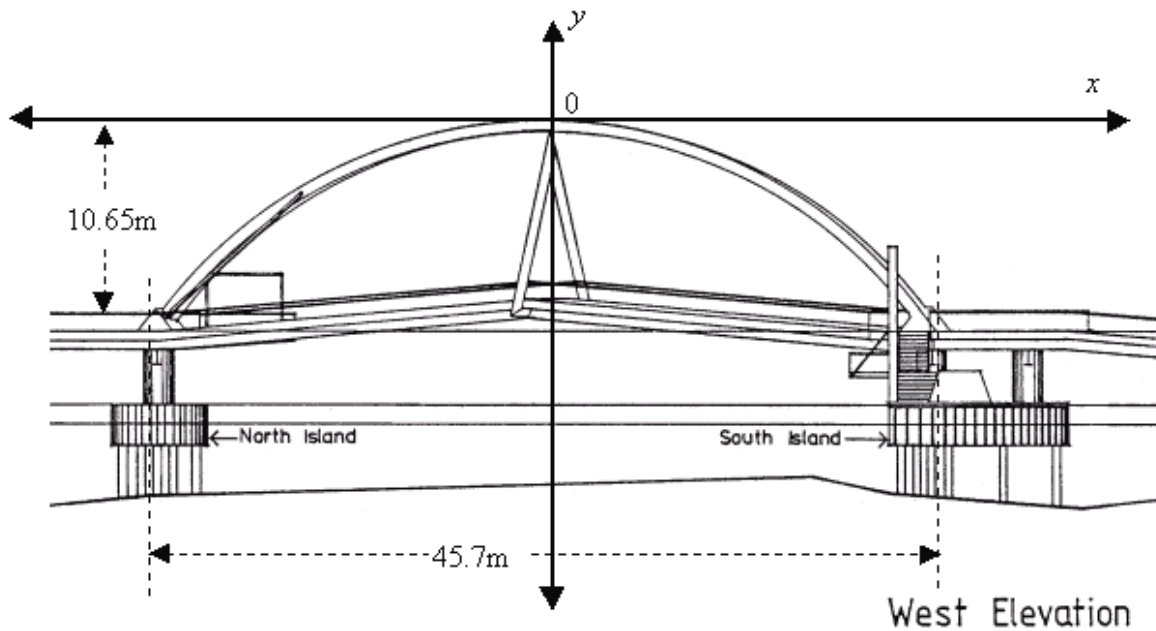
7. If there are two plantings of the clock each year, what is the total number of hours required for planting?

8. In addition to the planting, each of the four gardeners must also spend about two hours per week throughout the other 50 weeks of the year maintaining the clock and surrounding garden beds. Calculate the number of hours maintenance required.

9. What is the total number of hours per year required for planting and maintaining the clock and surrounding garden beds?

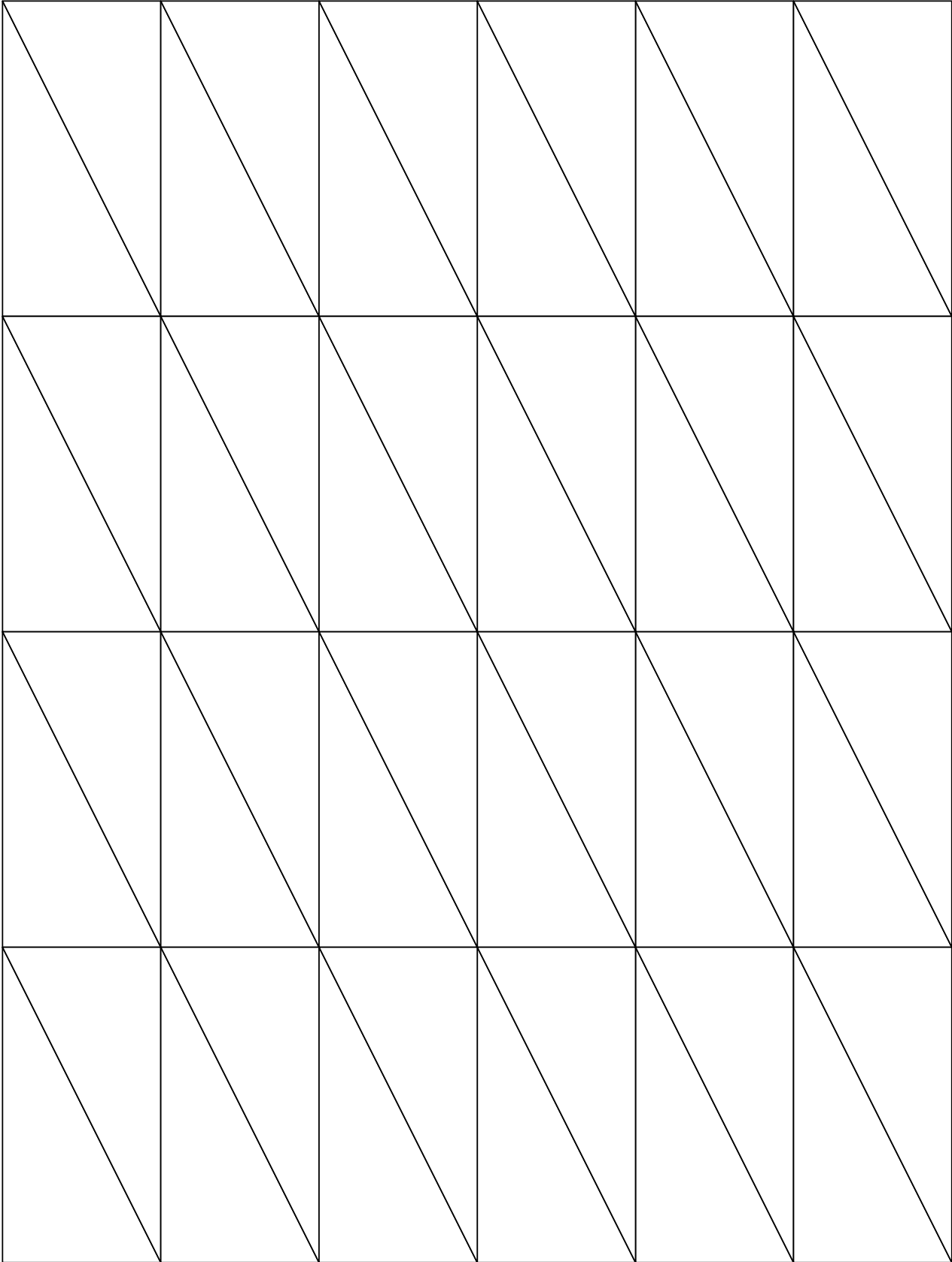
PEDESTRIAN BRIDGE AT SOUTHGATE

1. The pedestrian bridge is a parabola. Using the coordinate axes and dimensions shown on the architects' drawing of the pedestrian bridge, substitute into the equation $y = kx^2$ to determine the value of k . What is the equation for the parabolic shape of the bridge with the origin in the position shown?



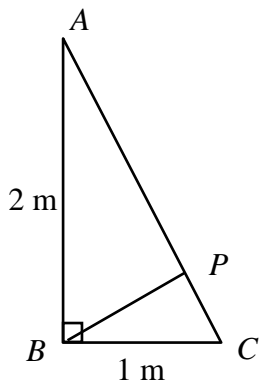
2. Use a graphics calculator to graph your equation. (For a TI-83™ calculator, press WINDOW and set Xmin at -22.85 and Xmax at 22.85. Press ZOOM and select 5: ZSquare so the shape is not distorted.)

FEDERATION SQUARE



Federation Square tiles

- Using two of the triangle tiles, make each of the following shapes:
 - A rectangle
 - A kite
 - An isosceles triangle
 - A different isosceles triangle
- Using four of the triangle tiles, make each of the following shapes:
 - A square
 - A rectangle
 - A rhombus
 - A parallelogram
- Using eight of the triangles, make each of the following shapes:
 - An isosceles triangle
 - A kite
- List all the rhombus properties which you can identify when four tiles are formed into a rhombus.
- In $\triangle ABC$, P is a point such that $\angle APB$ is a right-angle. Prove that $\triangle BPC$ is similar to $\triangle ABC$.



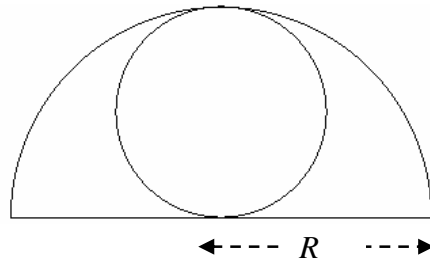
- Calculate the lengths of BP and PC (leave answers in surd form).
- Calculate the size of $\angle BAC$ and $\angle ACB$ to the nearest minute.

ST. PAUL'S CATHEDRAL

- Look at the windows along the Flinders Street and Swanston Street sides of St. Paul's Cathedral. List all the regular polygons that are used as the basis of circle patterns.
- Use pencil, ruler and compass, or computer dynamic geometry software such as Cabri Geometry™, to construct an equilateral arch and one of the trefoils or quatrefoils.

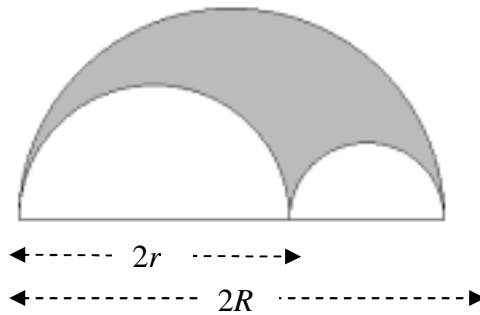
MELBOURNE TOWN HALL

1. If the radius of the semicircle below is R , show that the area of the circle is half the area of the semicircle.



Semicircle with inscribed circle.

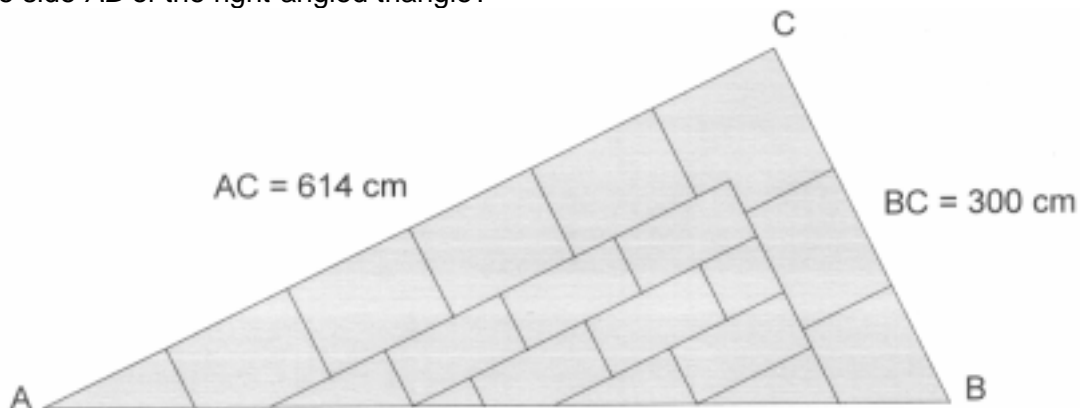
2. Find the area of the arbelos in terms of R and r .



The Shoemaker's Knife or Arbelos.

ARCHITECTURAL FRAGMENT

1. The 'roof' of *Architectural Fragment* is also a right-angled triangle. What name is given to the side AB of the right-angled triangle?

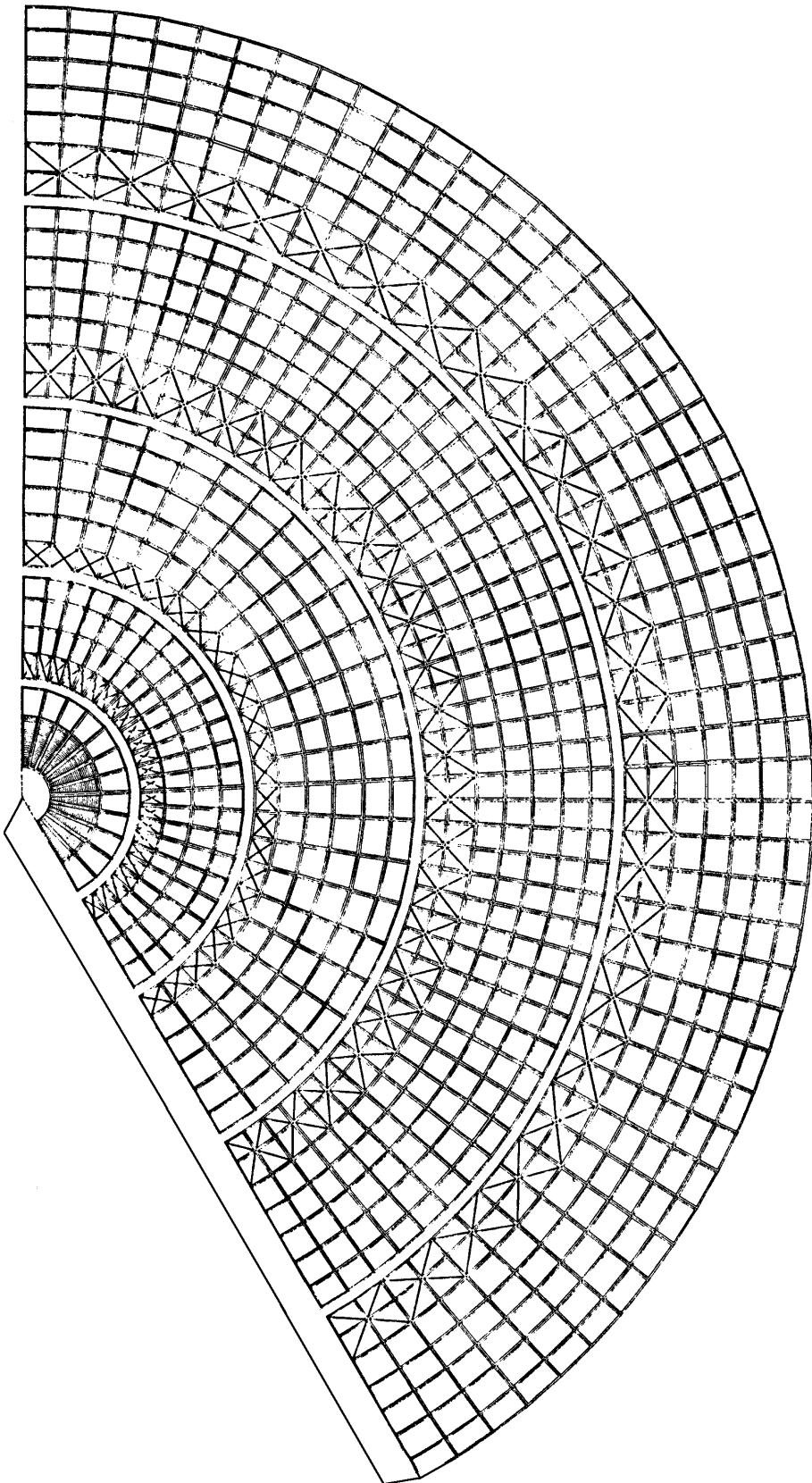


Back elevation ("roof") of Architectural Fragment.

1. Use Pythagoras' Theorem to calculate the length of AB .
2. Calculate the area of the triangle ABC .
3. Measure or calculate angle CAB and angle CBA .
4. How many similar triangles can you see in this triangular face?

MELBOURNE CENTRAL CONE

1. Here is the architect's drawing of the net of the Melbourne Central cone. Copy and cut out the net and use the flap to glue it into a cone shape.



2. Using the architect's drawing of Melbourne Central (Figure 6.32) to find the vertical height of the cone:

Height of cone on drawing = cm
Scale: 1 cm is equivalent to m
Height of actual cone = m

3. Using the architect's drawing, find the radius of the actual cone:

Diameter of cone on drawing = cm
Scale: 1 cm is equivalent to m
Diameter of actual cone = m
Radius of actual cone = m

4. Calculate the circumference and area of the base of the cone. Why would this area be of interest to the developers?

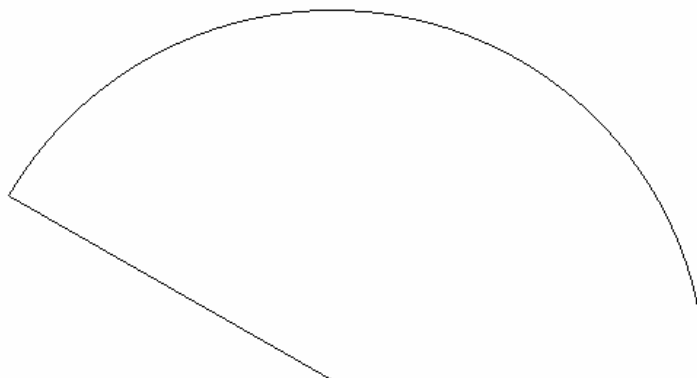
5. What is the volume of the cone?

$$\text{Volume of cone} = \frac{1}{3} \pi \times (\text{radius})^2 \times \text{height}$$

6. What angle does the sloping edge of the cone make with the horizontal? (Measure the angle on the architect's drawing (Figure 6.32) or use trigonometry to calculate the angle).

7. What sort of triangle is the vertical section through the cone?

8. The developers who constructed Melbourne Central would have needed an estimate of the cost of glazing the cone (that is, filling in all the spaces between the steel with glass). To do this they needed to know the curved surface area of the cone. Calculate the area of the curved surface of the cone. Label on the diagram below any additional measurements you made or calculated from the architect's drawings.



Sector of circle representing the curved surface of the cone.

THE SHOT TOWER

1. When an object is dropped it falls, gaining speed due to the effect of gravity. The acceleration due to gravity represents an increase in speed of 9.8 metre/sec every second. We write this as $g \text{ m/sec}^2$.

If s metres is the distance fallen, t seconds is the time taken to fall this distance and $g \text{ m/sec}^2$ is the acceleration due to gravity, then

$$s = \frac{1}{2} g t^2$$

When the molten lead was dropped from the top of the shot tower, it fell about 50 metres (check this on the architect's drawing in Figure 6.32). Using $s = 50$ and $g = 9.8$, find the time it took to fall.

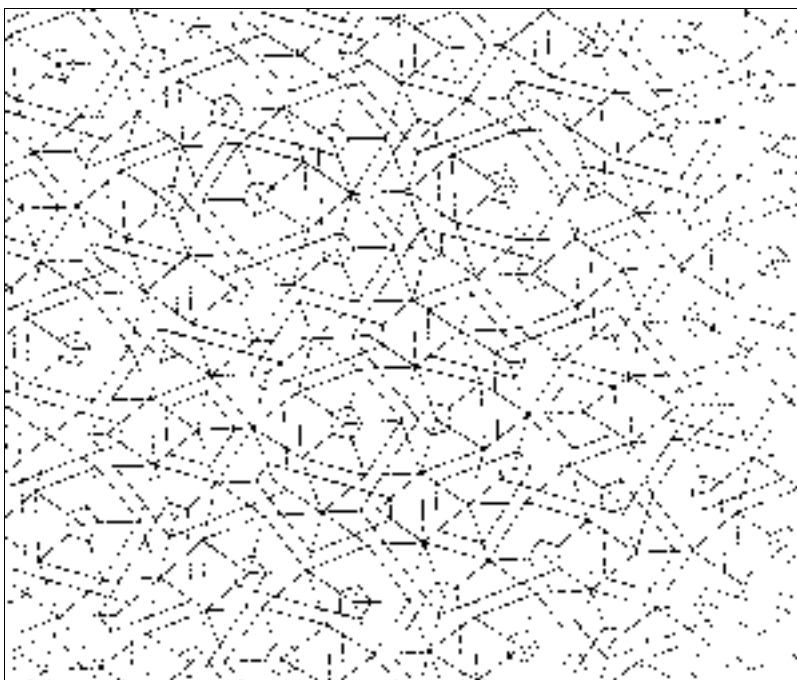
2. The speed, v metre/sec, of the lead after falling for t seconds is given by

$$v = g t$$

Using $g = 9.8$ and your value of t which you calculated above, calculate the speed of the lead shot just before it entered the water.

PENROSE TILES AT STOREY HALL, RMIT

1. How many different rotations of the fat and thin tiles occur?
2. Through how many degrees are the tiles rotated for each new rotation?
3. Cut out some of the fat and thin rhombus shapes and investigate how they tessellate.
4. Using Figure 6.37 to help you, colour the bands of green over the fat and thin rhombuses. This will give you the pattern that is superimposed over the rhombuses in Storey Hall:
5. Calculate the sizes of angles in the kite and dart in Figure 6.40.



Penrose rhombuses with superimposed pattern.

(Drawing supplied by the Architects: Ashton Raggatt McDougall).

Solutions

FLORAL CLOCK

1. Inner circle: $\pi \times 2.5^2 = 19.6 \text{ m}^2$; outer section: $\pi \times (4^2 - 2.5^2) = 30.6 \text{ m}^2$
2. $4000 \text{ plants} / 19.6 \text{ m}^2 = 204 \text{ plants/m}^2$
3. Estimate size of small circle at centre of design and use this as basis for estimating other areas.

■ ≈ 500 □ ≈ 1000 □ ≈ 1500 □ ≈ 1000

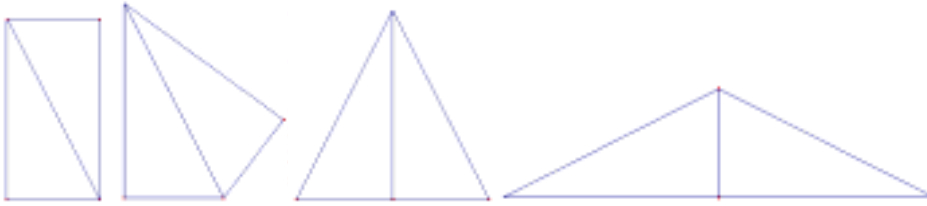
4. $196,000/4000 = 49 \text{ cm}^2/\text{plant}$, i.e., approx. $7 \text{ cm} \times 7 \text{ cm}$
5. $(7000/144) \times 100/90 \times \$35 \approx \$1890$
6. $4 \times 4 \times 7.5 = 120 \text{ hours}$
7. 240 hours
8. 400 hours
9. $240 + 400 = 640 \text{ hours}$

PEDESTRIAN BRIDGE AT SOUTHGATE

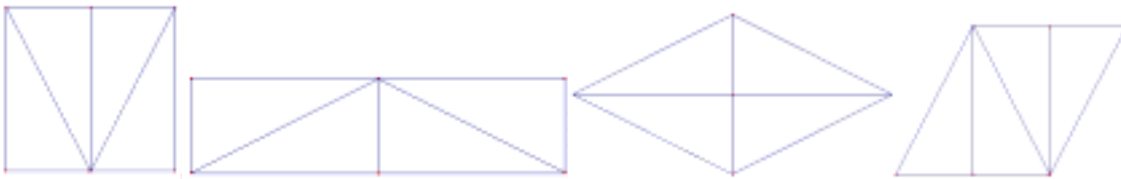
1. Using the point (22.85, -10.65),
 $y = kx^2$
 $-10.65 = 22.85^2 \times k$
 $k = -0.0204$
 $y = -0.0204 x^2$

FEDERATION SQUARE

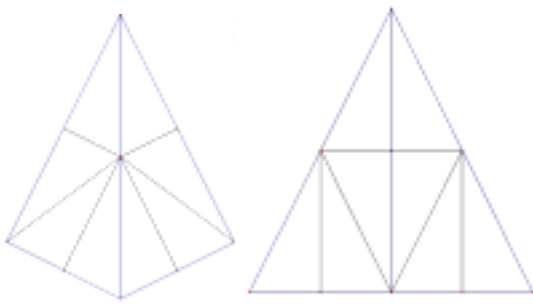
1.



2.



3.



4. All sides equal

Diagonals bisect each other

Diagonals intersect at right angles

Opposite angles equal

Opposite sides parallel

5. In $\triangle BPC$ and $\triangle ABC$:

$\angle ACB$ is common

$\angle ACB = \angle BPC = 90^\circ$

$\therefore \angle PBC = \angle BAC$

$\therefore \triangle BPC \sim \triangle ABC$

6.

$$\frac{\overline{PC}}{1} = \frac{1}{\sqrt{5}}$$

$$\therefore \overline{PC} = \frac{\sqrt{5}}{5}$$

$$\overline{BP} = \frac{2\sqrt{5}}{5}$$

7. $\tan \angle BAC = 2$

$\therefore \angle BAC \approx 63^\circ 26'$

$\angle ACB \approx 26^\circ 34'$

ST. PAUL'S CATHEDRAL

1. Equilateral triangle, square, pentagon, hexagon, heptagon.

MELBOURNE TOWN HALL

1. Area of semicircle = $\frac{\pi R^2}{2}$ Radius of circle = $\frac{R}{2}$

$$\text{Area of circle} = \pi \left(\frac{R}{2}\right)^2 = \frac{\pi R^2}{4} = \text{Half area of semicircle}$$

2. Area of arbelos = Area of large semicircle - area of two small semicircles

$$\begin{aligned} & \frac{\pi R^2}{2} - \frac{\pi(R-r)^2}{2} - \frac{\pi r^2}{2} \\ &= \frac{\pi R^2 - \pi R^2 + 2\pi Rr - \pi r^2 - \pi r^2}{2} \\ &= \frac{2\pi Rr - 2\pi r^2}{2} \\ &= \pi r(R-r) \end{aligned}$$

ARCHITECTURAL FRAGMENT

1. Hypotenuse

$$2. \sqrt{614^2 + 300^2} = 683.4 \text{ cm}$$

$$3. 614 \times 300 \div 2 = 92100 \text{ cm}^2 (9.21 \text{ m}^2)$$

$$4. \tan A = 300/614$$

$$\therefore \angle CAB = 26^\circ; \angle CBA = 64^\circ$$

5. Nine similar triangles

MELBOURNE CENTRAL CONE

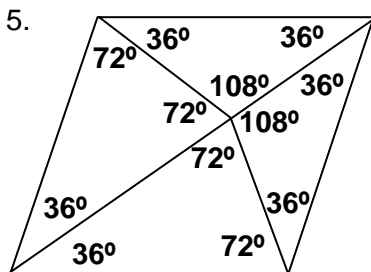
- Scale: 5.25 cm: 30 m \equiv 1 cm: 5.7 m
Height on drawing: 8.5 cm
Actual height = $8.5 \times 5.7 = 48.5$ m
- Diameter on drawing: 7.6 cm.
Actual diameter = $7.6 \times 5.7 = 43$ m
Radius: 21.5 m
- Circumference: $2\pi \times 21.5 \approx 135$ m,
Area: $\pi \times 21.5^2 \approx 1450$ m²
- Volume: $\frac{1}{3}\pi \times 21.6^2 \times 47.9$
 ≈ 23500 m³
- $\tan^{-1}(48.5/21.5) \approx 66^\circ$
- Isosceles triangle.
- Slant edge of cone = $\sqrt{21.5^2 + 48.5^2} \approx 53$ m
Angle of sector = 150°
Area = $\pi \times 53^2 \times 150 \div 360 \approx 3680$ m²

SHOT TOWER

- $s = \frac{1}{2}gt^2$ $50 = \frac{1}{2} \times 9.8 \times t^2$
 $\therefore t \approx 2.17$ seconds
 $v = gt = 9.8 \times 2.17 \approx 21$ m/sec

PENROSE TILES AT STOREY HALL

- Five rotations.
- The tiles are rotated through 72°



Thank you to Jill Vincent and MAV for granting permission to publish these extracts. The full version of Jill Vincent's *Shrine to University, A geometry journey along St.Kilda Road and Swanston Street* 2nd edition 2003 is available from:

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