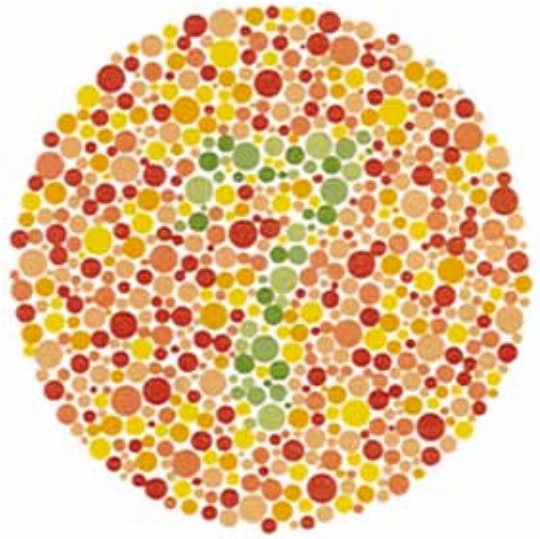
MathsBites by Clifford the Dog

Deranged arrangements!

Handing back tests

The result of a test for colour blindness on a small group of people is handed back to them randomly. What is the probability that no person receives their own test result? If there are only two people in the group, this is 50% (either you receive your own test result back or you don't). How many different ways could the test result be handed back for three people? What is the probability that no person receives in a group of three receives their own test result back?



Activity: Find this probability that no person gets their own test result back for a group of four people, see for example: http://demonstrations.wolfram.com/DerangementDiagrams/

Derangements

Given a set with its elements in some order a *derangement* is a re-arrangement (permutation) such that no element remains in its original position. For example if a group of ten people left their jackets in a restaurant cloakroom, a derangement would have occurred if no one got their own jacket handed back to them when they left, for example: http://en.wikipedia.org/wiki/Derangement

If $d(n)$ represents the numbers of derangements for a set of size n, where n is a positive integer, then one can find $d(n)$ for small values for n by inspection of arrangements of the elements of the set. Systematically list of all possible arrangements of $\{A, B, C, D, E\}$ and hence find the proportion of these that are derangements, see, for example: http://mathworld.wolfram.com/Derangement.html

Activity: Show that $d(1) = 0$, $d(2) = 1$, and $d(n) = (n - 1)[d(n - 1) + d(n - 2)]$. Use this recursion relation to find the probability that if you randomly handed back the next class test to students, no student will receive their own test. It can be shown that for a *large* group this probability is close to $1/e \approx 37\%$.

See for example, *Hat checks and derangements*: http://www.youtube.com/watch?v=6XeCheL9XnI