MathsBites by Clifford the Dog

Some forms of fractions

When does a fraction have a terminating decimal?

The decimal expansion of a fraction $\frac{m}{n}$ is obtained by calculating $m \div n$. For example the decimal expansion of $\frac{3}{8}$ can be obtained by calculating:

$$
\begin{array}{r|lllll}
8 & 3 & . & {}^{3}0 & {}^{6}0 & {}^{4}0 \\
\hline
 & 0 & . & 3 & 7 & 5
\end{array}
$$

This is a *finite* or *terminating* decimal expansion.

Activity: calculate the decimal expansions of $\frac{1}{n}$ for n from 1 to 10 and hence calculate the decimal expansions of $\frac{m}{n}$ for m from 1 to $n - 1$. Which of these have a terminating decimal expansion?

The following table shows decimal expansions for $\frac{1}{11}$ to $\frac{1}{20}$

1/11	0.090909090909090909090909090909090909090909090909 ...
1/12	0.083333333333333333333333333333333333333333333333 ...
1/13	0.076923076923076923076923076923076923076923076923 ...
1/14	0.071428571428571428571428571428571428571428571428 ...
1/15	0.066666666666666666666666666666666666666666666666 ...
1/16	0.062500000000000000000000000000000000000000000000 ...
1/17	0.058823529411764705882352941176470588235294117647 05 ...
1/18	0.055555555555555555555555555555555555555555555555 ...
1/19	0.052631578947368421052631578947368421052631578947 36 ...
1/20	0.050000000000000000000000000000000000000000000000 ...

Activity: use this table, and others like it, to determine when a fraction $\frac{m}{n}$ has a terminating or a recurring decimal expansion. By considering the denominator as a product of powers of prime numbers, explain why this is the case. (See, for example, http://mathworld.wolfram.com/DecimalExpansion.html). Is there any relation between period of recurrence and m or n?

The Eye of Horus

Ancient Egyptians used a symbolic form called the *Eye of Horus* to represent various fractions, for example, the eyebrow represented $\frac{1}{8}$, and the whole eye represented $\frac{63}{64}$.

Activity: research what these fractions were and how they arose and were used. (See, for example: http://en.wikipedia.org/wiki/Eye_of_Horus and www.greatscott.com/hiero/eye.html