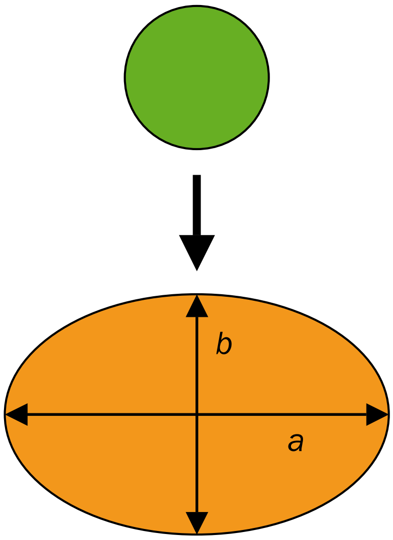
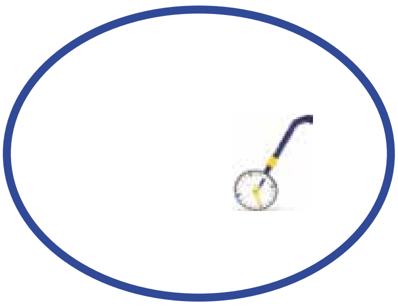
MathsBites by Clifford the Dog

Filling and fencing an ellipse

Filling an ellipse

A unit square has side length 1 and area 1. If its horizontal and vertical dimensions are enlarged by factors of a and b respectively, then area of the corresponding rectangle is $1 \times a \times b$. The unit circle $x^2 + y^2 = 1$ has radius 1, perimeter 2π and area π. If its horizontal and vertical dimensions are both enlarged by a factor r, then this unit circle becomes the circle $(x/r)^2 + (y/r)^2 = 1$ or $x^2 + y^2 = r^2$ and has area $\pi \times r \times r = \pi r^2$. If a unit circle's horizontal and vertical dimensions are enlarged by factors a and b respectively, then it becomes the ellipse $x^2/a^2 + y^2/b^2 = 1$ with area $\pi \times a \times b$ see: www.mathsisfun.com/geometry/ellipse.html; http://mathworld.wolfram.com/Ellipse.html.





$$\text{Perimeter} = 4\int_0^a \sqrt{1+(f'(x))^2}\,dx$$

where $f(x) = b\sqrt{1-\dfrac{x^2}{a^2}}$

Fencing an ellipse

The perimeter of a circle of radius r is given by $\pi \times (r + r) = 2\pi r$. What is the perimeter of an ellipse? Perhaps surprisingly, there is no simple exact formula. The formula $\pi \times (a + b)$ analogous to that for a circle, provides a rough approximation which is reasonable when the ellipse is nearly circular. Another approximation, by Euler, is $\pi \times \sqrt{(2(a^2 + b^2))}$. This is accurate to about 5% when the ellipse is not too 'squashed'. There are other approximation formulas and some exact series expressions, the perimeter of an ellipse specified as cartesian relation is typically calculated using numerical integration for arc lengths, see: www.mathsisfun.com/geometry/ellipse-perimeter.html; http://home.att.net/~numericana/answer/ellipse.htm.

Laying out an oval

How are football ovals actually laid out? Are they ellipses? How does the shape affect the pocket positions? What method would council ground-staff use to set up a new oval today? 30 years ago? What rules of thumb would they use to calculate perimeter and area?

See: http://en.wikipedia.org/wiki/Australian_rules_football_positions