MathsBites by Clifford the Dog

The square root of two

Quick square

A two digit number can be quickly squared: $15^2 = (10 + 5)^2 = 100 + 50 + 50 + 25 = 225$ (see diagram). Using place value knowledge $1.5^2 = {}^{15}/_{10} \times {}^{15}/_{10} = {}^{15^2}/_{100} = 2.25$. Try this with some other numbers such as 14^2, 27^2 and 85^2. Use a similar method to calculate 140^2, 150^2, 270^2 and 850^2.

×	10	+5
10	100	50
+5	50	25

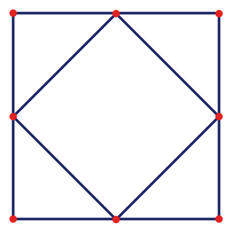
x	x^2
1.45	2.1025
1.44	2.0736
1.43	2.0449
1.42	2.0164
1.41	1.9881

If $x^2 = 2$, what is x?

We know that x must lie between 1.4 and 1.5 since $14^2 = 196$ and $15^2 = 225$ so $1.4^2 = 1.96$ and $1.5^2 = 2.25$. The table shows part of a systematic evaluation process. Extend this to obtain a value for x such that x^2 rounds to 2, correct to four decimal places. Explain why it is not possible to find a finite decimal of the form $1.a_1a_2a_3 \ldots a_n$ where a_n is non-zero, such that $x^2 = 2$.

A square with side length exactly root two

The midpoints of a square with side length 2 are joined to form a new shape. Explain why the new shape is also a square, and (without using Pythagoras theorem) why it has side length $\sqrt{2}$.



n	x_n
0	2.5
1	2.4
2	2.416666...
3	2.413793...
4	2.414285...
5	2.414201...
6	2.414215...
7	2.414213...

Solving $x^2 - 2x - 1 = 0$ in two different ways

Firstly: $x^2 - 2x - 1 = 0$ implies $(x - 1)^2 - 2 = 0$ and so $x = 1 - \sqrt{2}$ or $x = 1 + \sqrt{2}$. Secondly: $x^2 - 2x - 1 = 0$ implies $x^2 = 2x + 1$ and $x = 2 + {}^{1}/_{x}$ for $x \neq 0$. Re-write this as a recurrence relation: $x_{n+1} = 2 + {}^{1}/_{x_n}$ and choose an initial value, such as $x_0 = 2.5$. Values of the sequence $x_n - 1$ approximate $\sqrt{2}$. This can be implemented on a calculator using the recursion function, or step by step using the 'Ans' memory and repeatedly calculating $2 + ({}^{1}/_{\text{Ans}})$. After seven iterations this gives $\sqrt{2} \approx 2.41421 - 1 = 1.41421$. Find the percentage error.

Decimal expansion of the square root of two

An initial sequence of the infinite non-recurring decimal expansion of the square root of 2 is:

1.4142135623730950488016887242096980785696718753769480731766797379907
32478462107038850387534327641572735013846230912297024924836055850737
21264412149709993583141322266592750.

Do all the digits {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} in this initial sequence occur with approximately equal frequency?