MathsBites by Clifford the Dog

Some fixed point iterations

Applying a function again and again and again and again and again …

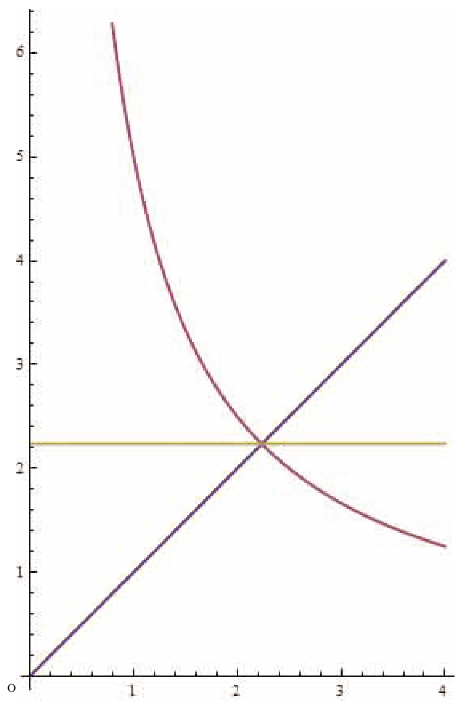
Russian Matroshka dolls comprise a nested set of dolls. A similar thing can be done with functions, and is called fixed point iteration. http://en.wikipedia.org/wiki/Fixed-point_iteration Start with an initial value such as $x = 2$, apply a function, for example $f(x) = x^2$ and repeatedly feed the output into the function as the next input. In this case: $2 \rightarrow 4 \rightarrow 16 \rightarrow 256 \rightarrow \ldots$ This generates the sequence of values $\{2, f(2), f(f(2)), f(f(f(2))) \ldots \}$.

Generate similar sequences for some other starting values. What is the general sequence for f applied to an initial value of a?

http://en.wikipedia.org/wiki/File:Russian-Matroshka_no_bg.jpg (creative commons)

Activity: Explore what happens when f is the cosine function, and use the graphs of $y = \cos x$ and $y = x$ with a cobweb diagram to illustrate this behaviour. See, for example, http://elishapeterson.wikidot.com/technotes:geogebra-cobwebs

Square roots as fixed point iterations



Square roots can also be evaluated using fixed point iteration.

The function that averages x and $\frac{k}{x}$ can be used to provide a fixed point iteration that approximates $\sqrt{k}$ given a suitable starting value.

For example, if $k = 5$, then starting with $a = 2$ where $f(x) = \frac{1}{2}\left(\frac{5}{x} + x\right)$ produces the sequence $\{2, 2.25, 2.2361, 2.23607, 2.23607\ldots\}$.

The graph on the left shows $y = x$, $y = \frac{5}{x}$ and $y = \sqrt{5}$ on the same set of axes.

See also www.youtube.com/watch?v=OLqdJMjzib8

Activity: Let $f(x) = \frac{1}{2}\left(\frac{k}{x} + x\right)$ Use the iteration expression $x = f(x)$ to show this leads to a solution for $x = \sqrt{k}$. Try this process for different values of a and k.