Meet the Assessor
Assessment Report for Exam 2
Statistics 2015 Total 80

• Highest score  80
• Lowest score   0
• n            15 634   (2014 15 557)
Scaling 2015

- Mean 33.8 (33.4) and SD 8.5 (8.4)
- 20 (21)
- 25 (28)
- 30 (34) -1
- 35 (40) -1
- 40 (45)
- 45 (49)
- 50 (51) +1

[Link to 2016 VCAA\Scaling 2015.pdf]
Grade Distribution

- vce_math_methods_cas_ga11.pdf
Multiple Choice 2015

- 95% answered Question 1 and 91% answered Question 6 correctly.

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Let $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 2\sin(3x) - 3$.
The period and range of this function are respectively

A. period $= \frac{2\pi}{3}$ and range $=[-5, -1]$

B. period $= \frac{2\pi}{3}$ and range $=[-2, 2]$

C. period $= \frac{\pi}{3}$ and range $=[-1, 5]$

D. period $= 3\pi$ and range $=[-1, 5]$

E. period $= 3\pi$ and range $=[-2, 2]$
Question 6

For the polynomial $P(x) = x^3 - ax^2 - 4x + 4$, $P(3) = 10$, the value of $a$ is

A. $-3$
B. $-1$
C. $1$
D. $3$
E. $10$
A Question 2 (B 42)  50%

Question 2
The inverse function of \( f : (-2, \infty) \rightarrow \mathbb{R}, f(x) = \frac{1}{\sqrt{x+2}} \) is

A. \( f^{-1} : \mathbb{R^+} \rightarrow \mathbb{R} \)
   \[ f^{-1}(x) = \frac{1}{x^2} - 2 \]

B. \( f^{-1} : \mathbb{R \setminus \{0\}} \rightarrow \mathbb{R} \)
   \[ f^{-1}(x) = \frac{1}{x^2} - 2 \]

C. \( f^{-1} : \mathbb{R^+} \rightarrow \mathbb{R} \)
   \[ f^{-1}(x) = \frac{1}{x^2} + 2 \]

D. \( f^{-1} : (-2, \infty) \rightarrow \mathbb{R} \)
   \[ f^{-1}(x) = x^2 + 2 \]

E. \( f^{-1} : (2, \infty) \rightarrow \mathbb{R} \)
   \[ f^{-1}(x) = \frac{1}{x^2 - 2} \]
A Question 2 (B 42)  50%

\[ f(x) = \frac{1}{\sqrt{x + 2}} \]

Let \( y = \frac{1}{\sqrt{x + 2}} \)

For inverse: swap \( x \) and \( y \)

\[ x = \frac{1}{\sqrt{y + 2}} \]

\[ x^2 = \frac{1}{y + 2} \]

\[ y = \frac{1}{x^2} - 2 \]

\[ f^{-1}(x) = \frac{1}{x^2} - 2 \]

Domain of \( f \) is 
\( x + 2 > 0, \ x > -2 \)

Range of \( f \) is \( R^+ \) which is the domain of \( f^{-1} \)

\[ f^{-1} : R^+ \rightarrow R, f^{-1}(x) = \frac{1}{x^2} - 2 \]
The rule for a function with the graph above could be

A. $y = -2(x + b)(x - c)^2(x - d)$

B. $y = 2(x + b)(x - c)^2(x - d)$

C. $y = -2(x - b)(x - c)^2(x - d)$

D. $y = 2(x - b)(x - c)(x - d)$

E. $y = -2(x - b)(x + c)^3(x + d)$
The rule for the graph is consistent with the form \( f(x) = a(x - b)(x - c)^2(x - d) \), where \( a \) is negative and could be \(-2\).

\[ f(x) = -2(x - b)(x - c)^2(x - d) \]

Note \( b \) is negative, for example if \( b = -1 \), the factor is \((x - (-1)) = (x + 1)\).
The graph of the probability density function of a continuous random variable, $X$, is shown below.

If $a > 2$, then $E(X)$ is equal to
A. 8  
B. 5  
C. 4  
D. 3  
E. 2

Solve $\int_{0}^{a} \left( \frac{1}{6} \right) dx = 1$, for $a$.

$a = 8$

$E(X) = \int_{2}^{8} \left( \frac{x}{6} \right) dx = 5$
OR

Area of rectangle above the $x$-axis equals 1.

So $\frac{1}{6}(a - 2) = 1$, $a = 8$.

For a uniform distribution, $E(X)$ must be midway between $x = 2$ and $x = 8$. Therefore $E(X) = 2 + 3 = 5$ or $E(X) = 8 - 3 = 5$. 
B Question 9 (C 21) 37%

\[
\text{solve } \int_{a}^{a} \frac{x}{6} \, dx = 1, a
\]

\[a = -4 \text{ or } a = 4\]
A Question 11 (D 32) 24%

Question 11
The transformation that maps the graph of \( y = \sqrt{8x^3 + 1} \) onto the graph of \( y = \sqrt{x^3 + 1} \) is a

A. dilation by a factor of 2 from the \( y \)-axis.
B. dilation by a factor of 2 from the \( x \)-axis.
C. dilation by a factor of \( \frac{1}{2} \) from the \( x \)-axis.
D. dilation by a factor of 8 from the \( y \)-axis.
E. dilation by a factor of \( \frac{1}{2} \) from the \( y \)-axis.

\[ y_1 = \sqrt{8x^3 + 1}, \quad y_2 = \sqrt{8\left(\frac{x}{2}\right)^3 + 1} = \sqrt{x^3 + 1} \]

Dilation by a factor of 2 from the \( y \)-axis. This can be shown by sketching the graphs of both functions. For example, the point with coordinates (1, 3) is transformed to (2, 3).
A Question 11 (D 32) 24%

\[ f_6(x) = \sqrt[8]{x^3 + 1} \]

\[ f_7(x) = \sqrt{x^3 + 1} \]
Question 16
Let \( f(x) = ax^m \) and \( g(x) = bx^n \), where \( a, b, m \) and \( n \) are positive integers. The domain of \( f = \) domain of \( g = \mathbb{R} \). If \( f'(x) \) is an antiderivative of \( g(x) \), then which one of the following must be true?

A. \( \frac{m}{n} \) is an integer

B. \( \frac{n}{m} \) is an integer

C. \( \frac{a}{b} \) is an integer

D. \( \frac{b}{a} \) is an integer

E. \( n - m = 2 \)
\[ f'(x) = \int (bx^n) \, dx = \frac{bx^{n+1}}{n+1} + c = \frac{bx^{n+1}}{n+1} \]

\[ f'(x) = amx^{m-1} \]

\[ n + 1 = m - 1, n = m - 2, \text{ (not E)} \]

\[ \frac{b}{n+1} = am, \quad \frac{b}{a} = m(n+1) = m(m-1) \]

Hence \( \frac{b}{a} \) is an integer as \( m \) is an integer, for example if \( m = 4 \), \( \frac{b}{a} = 4 \times 3 = 12 \).
Question 18

For which one of the following functions is the equation $|f(x + y) - f(x - y)| = 4\sqrt{f(x)f(y)}$ true for all $x \in \mathbb{R}$ and $y \in \mathbb{R}$?

A. $f(x) = x^2$

$$f(x) = x^2$$

$$|f(x + y) - f(x - y)| = |(x + y)^2 - (x - y)^2|$$

$$= |4xy|$$

$$= 4\sqrt{x^2y^2}$$

$$= 4\sqrt{f(x)f(y)}$$

B. $f(x) = |2x|$

C. $f(x) = e^x$

D. $f(x) = x^3$

E. $f(x) = x$
Question 21
The graphs of \( y = mx + c \) and \( y = ax^2 \) will have no points of intersection for all values of \( m, c \) and \( a \) such that

A. \( a > 0 \) and \( c > 0 \)

\[ mx + c = ax^2 \]
\[ ax^2 - mx - c = 0 \]

The discriminant will be negative for no real solutions.

\[ m^2 + 4ac < 0 \]

\( c < -\frac{m^2}{4a}, a > 0 \)

B. \( a > 0 \) and \( c < 0 \)

\( c > -\frac{m^2}{4a}, a < 0 \)

C. \( a > 0 \) and \( c > -\frac{m^2}{4a} \)

D. \( a < 0 \) and \( c > -\frac{m^2}{4a} \)

E. \( m > 0 \) and \( c > 0 \)
Question 22

The graphs of the functions with rules $f(x)$ and $g(x)$ are shown below.

Which one of the following best represents the graph of the function with rule $g(-f(x))$?
B Question 22 (D 26)  35%
B Question 22 (D 26) 35%

Make up equations for the functions, for example \( g(x) = \tan(x) \) and \( f(x) = -|x| \).

\[ g(-f(x)) = \tan(|x|) \]
Extended Answer

- Give an exact answer unless otherwise stated.
  \[ \frac{1}{3} = 0.3 \quad \text{OR} \quad \frac{1}{3} \]

- No calculator syntax.
- Show working for questions worth more than one mark. (Rule and answer)
- Work to more decimal places than the required answer.
- Use the variables that are given within the question. (SAC questions)
Extended Answer

• Reread questions.

• Take time when drawing graphs – scale axes, check if coordinates are required, one sharp line...

• Don’t assume steps in Show That questions.

• Use brackets correctly.

• Transcribe formulas correctly (reread the calculator).
Extended Answer

• Put units in the final answer.
• Check that the final answer makes sense.
Question 1a

Let \( f : \mathbb{R} \rightarrow \mathbb{R}, \quad f(x) = \frac{1}{5} (x - 2)^2 (5 - x) \). The point \( P \left( 1, \frac{4}{5} \right) \) is on the graph of \( f \), as shown below.

The tangent at \( P \) cuts the \( y \)-axis at \( S \) and the \( x \)-axis at \( Q \).

There are many variations. Some are shown below.

\[
\begin{align*}
    f(x) &= \frac{1}{5} (x - 2)^2 (5 - x) \\
    f'(x) &= \frac{2}{5} (5 - x)(x - 2) - \frac{1}{5} (x - 2)^2 \\
    \text{OR} \\
    f'(x) &= \frac{-3}{5} (x - 4)(x - 2) \\
    \text{OR} \\
    f'(x) &= \frac{-1}{5} (3x^2 - 18x + 24)
\end{align*}
\]

a. Write down the derivative \( f'(x) \) of \( f(x) \). 1 mark
b. i. Find the equation of the tangent to the graph of \( f \) at the point \( P\left(1, \frac{4}{5}\right) \).

The equation of the tangent at \( x = 1 \) is
\[
y = -\frac{9}{5}x + \frac{13}{5}
\]
OR
\[
y = \frac{1}{5}(13 - 9x)
\]
OR
\[
y = -1.8x + 2.6
\]
OR
\[
y = \frac{13}{5} - \frac{9}{5}x
\]
Question 1bii

ii. Find the coordinates of points \( Q \) and \( S \).

\[5y + 9x = 13\]

Let \( x = 0, \ y = \frac{13}{5}, \ S \left( 0, \frac{13}{5} \right) \) or \( (0, 2.6) \)

Let \( y = 0, \ x = \frac{13}{9}, \ Q \left( \frac{13}{9}, 0 \right) \) or \( (1.4, 0) \)
c. Find the distance $PS$ and express it in the form $\frac{\sqrt{b}}{c}$, where $b$ and $c$ are positive integers.

$S \left(0, \frac{13}{5}\right)$ and $P \left(1, \frac{4}{5}\right)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$dPS = \sqrt{\left(\frac{13}{5} - \frac{4}{5}\right)^2 + (0 - 1)^2}$

$= \frac{\sqrt{106}}{5}$
d. Find the area of the shaded region in the graph above.

Solve \( f(x) = \frac{1}{5}(13 - 9x) \) for \( x \).

\[
x = 1 \quad \text{or} \quad x = 7
\]

Area is given by

\[
\int_{1}^{7} \left( f(x) - \left(-\frac{9}{5}x + \frac{13}{5}\right) \right) dx
\]

\[
= \frac{108}{5} \quad \text{or} \quad 21.6
\]
Question 1d

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<td>25</td>
<td>14</td>
<td>7</td>
<td>54</td>
<td>1.9</td>
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\[
\int_{1}^{7} \left( f(x) - \left( -\frac{9}{5} \cdot x + \frac{13}{5} \right) \right) dx = \frac{108}{5} \\
\text{x=1 or x=7}
\]
Question 2

Question 2 (14 marks)
A city is located on a river that runs through a gorge.
The gorge is 80 m across, 40 m high on one side and 30 m high on the other side.
A bridge is to be built that crosses the river and the gorge.
A diagram for the design of the bridge is shown below.
Question 2a

The main frame of the bridge has the shape of a parabola. The parabolic frame is modelled by 
\[ y = 60 - \frac{3}{80} x^2 \]  
and is connected to concrete pads at \( B(40, 0) \) and \( A(-40, 0) \).

The road across the gorge is modelled by a cubic polynomial function.

a. Find the angle, \( \theta \), between the tangent to the parabolic frame and the horizontal at the point \( A(-40, 0) \) to the nearest degree.

Let  \[ g(x) = y = 60 - \frac{3}{80} x^2 \]

\[ g'(x) = -\frac{3}{40} x \]

\[ g'(-40) = 3 \]

\[ \tan(\theta) = 3 \]

\[ \theta = 71.56... = 72^\circ \]  to the nearest degree
Question 2a

Define \( g(x) = 60 - \frac{3}{80} \cdot x^2 \)

\[
\frac{d}{dx} \left( g(x) \right) \bigg|_{x=-40} = -40 \\
(tan^{-1}(3)) \rightarrow DMS = 71^\circ 33' 54.1842'' \\
\frac{tan^{-1}(3) \cdot 180}{\pi} = 71.5651
\]
The road from X to Y across the gorge has gradient zero at X(−40, 40) and at Y(40, 30), and has equation \( y = \frac{x^3}{25600} - \frac{3x}{16} + 35 \).

b. Find the maximum downwards slope of the road. Give your answer in the form \( -\frac{m}{n} \) where \( m \) and \( n \) are positive integers.

Let \( h(x) = y = \frac{x^3}{25600} - \frac{3x}{16} + 35 \)

\( h'(x) = \frac{3x^2}{25600} - \frac{3}{16} \)

The downward slope is greatest at \( x = 0 \).

\( h'(0) = -\frac{3}{16} \)

Maximum downward slope is \( -\frac{3}{16} \).
Question 2c

Two vertical supporting columns, \(MN\) and \(PQ\), connect the road with the parabolic frame. The supporting column, \(MN\), is at the point where the vertical distance between the road and the parabolic frame is a maximum.

c. Find the coordinates \((u, v)\) of the point \(M\), stating your answers correct to two decimal places.

\[
d(x) = g(x) - h(x)
\]

Solve \(d'(x) = 0\) for \(x\).

\[
x = 2.4903..., h(2.49...) = 34.5337...
\]

\(M\) (2.49, 34.53) correct to two decimal places.
The second supporting column, $PQ$, has its lowest point at $P(-u, w)$.

d. Find, correct to two decimal places, the value of $w$ and the lengths of the supporting columns $MN$ and $PQ$.

$h(-2.4903...) = 35.4663...$  
$w = 35.47$

$PQ = g(-2.4903..) - h(-2.4903..)$

$= 24.3011... = 24.30$ correct to two decimal places

$MN = g(2.4903...) - h(2.4903...)$

$= 25.2338... = 25.23$ correct to two decimal places
Question 2e

For the opening of the bridge, a banner is erected on the bridge, as shown by the shaded region in the diagram below.

e. Find the $x$-coordinates, correct to two decimal places, of $E$ and $F$, the points at which the road meets the parabolic frame of the bridge.
Question 2e

Solve \( f(x) = g(x) \) for \( x \).
At \( E \), \( x = -23.7068\ldots = -23.71 \) correct to two decimal places.
At \( F \), \( x = 27.9963\ldots = 28.00 \) correct to two decimal places.
f. Find the area of the banner (shaded region), giving your answer to the nearest square metre.

\[
\text{Area} = \int_{-23.706}^{27.996} (f(x) - g(x)) \, dx
\]

= 869.619... = 870 m\(^2\) to the nearest m\(^2\)
Question 3ai

Question 3 (11 marks)
Mani is a fruit grower. After his oranges have been picked, they are sorted by a machine, according to size. Oranges classified as medium are sold to fruit shops and the remainder are made into orange juice.

The distribution of the diameter, in centimetres, of medium oranges is modelled by a continuous random variable, \( X \), with probability density function

\[
f(x) = \begin{cases} 
\frac{3}{4} (x-6)^2 (8-x) & \text{if } 6 \leq x \leq 8 \\
0 & \text{otherwise}
\end{cases}
\]

a.  
ii. Find the probability that a randomly selected medium orange has a diameter greater than 7 cm.

\[
\Pr(X>7) = \int_{7}^{8} f(x) \, dx
\]

\[
= \frac{11}{16} \text{ or } 0.6875
\]
Question 3aii

ii. Mani randomly selects three medium oranges.

Find the probability that exactly one of the oranges has a diameter greater than 7 cm.
Express the answer in the form \( \frac{a}{b} \), where \( a \) and \( b \) are positive integers.

\[
Y \sim Bi \left( 3, \frac{11}{16} \right)
\]

\[
\Pr(Y = 1) = 3 \times \frac{11}{16} \times \left( \frac{5}{16} \right)^2
\]

\[
= \frac{825}{4096}
\]
Question 3b

b. Find the mean diameter of medium oranges, in centimetres.

\[ E(X) = \int_{6}^{8} (x \times f(x)) \, dx \]
\[ = \frac{36}{5} \]
Question 3c

For oranges classified as large, the quantity of juice obtained from each orange is a normally distributed random variable with a mean of 74 mL and a standard deviation of 9 mL.

c. What is the probability, correct to three decimal places, that a randomly selected large orange produces less than 85 mL of juice, given that it produces more than 74 mL of juice?

\[ \text{Oranges } \sim N(74, 9^2) \]

\[
\Pr(O < 85 \mid O > 74) = \frac{\Pr(O < 85 \cap O > 74)}{\Pr(O > 74)} = \frac{\Pr(74 < O < 85)}{\Pr(O > 74)} = 0.38918... = \frac{0.38918...}{0.5} = 0.7783... = 0.778 \text{ correct to three decimal places}
\]
Question 3di

Mani also grows lemons, which are sold to a food factory. When a truckload of lemons arrives at the food factory, the manager randomly selects and weighs four lemons from the load. If one or more of these lemons is underweight, the load is rejected. Otherwise it is accepted.

It is known that 3% of Mani’s lemons are underweight.

d.  
   i. Find the probability that a particular load of lemons will be rejected. Express the answer correct to four decimal places.

\[
Pr(\text{lemons } \geq 1) = 1 - Pr(\text{lemons } = 0)
\]

\[
= 1 - (0.97)^4
\]

\[
= 0.114707...
\]

\[
= 0.1147 \text{ correct to four decimal places}
\]
ii. Suppose that instead of selecting only four lemons, \( n \) lemons are selected at random from a particular load.

Find the smallest integer value of \( n \) such that the probability of at least one lemon being underweight exceeds 0.5

\[
\Pr (X \geq 1) > 0.5 \\
1 - (0.97)^n > 0.5 \\
n > 22.7566.. \\
n = 23
\]
Question 4 (9 marks)

An electronics company is designing a new logo, based initially on the graphs of the functions

\[ f(x) = 2 \sin(x) \quad \text{and} \quad g(x) = \frac{1}{2} \sin(2x), \text{ for } 0 \leq x \leq 2\pi. \]

These graphs are shown in the diagram below, in which the measurements in the \( x \) and \( y \) directions are in metres.
Question 4a

The logo is to be painted onto a large sign, with the area enclosed by the graphs of the two functions (shaded in the diagram) to be painted red.

a. The total area of the shaded regions, in square metres, can be calculated as \( a \int_0^\pi \sin(x) \, dx \).

What is the value of \( a \)?

\[
\text{Total area} = 2 \int_0^\pi (2 \sin(x)) \, dx
\]
\[
= 4 \int_0^\pi (\sin(x)) \, dx
\]
\[
a = 4
\]
Question 4b

The electronics company considers changing the circular functions used in the design of the logo. Its next attempt uses the graphs of the functions \( f(x) = 2\sin(x) \) and \( h(x) = \frac{1}{3}\sin(3x) \), for \( 0 \leq x \leq 2\pi \).

b. On the axes below, the graph of \( y = f(x) \) has been drawn.

On the same axes, draw the graph of \( y = h(x) \).
c. State a sequence of two transformations that maps the graph of \( y = f(x) \) to the graph of \( y = h(x) \).

The sequence of two transformations that map the graph of \( f(x) = 2\sin(x) \) to the graph of \( h(x) = \frac{1}{3}\sin(3x) \) is

- a dilation by a factor of \( \frac{1}{6} \) from the \( x \)-axis
- a dilation by a factor of \( \frac{1}{3} \) from the \( y \)-axis
Question 4di

The electronics company now considers using the graphs of the functions $k(x) = m \sin(x)$ and $q(x) = \frac{1}{n} \sin(nx)$, where $m$ and $n$ are positive integers with $m \geq 2$ and $0 \leq x \leq 2\pi$.

d. i. Find the area enclosed by the graphs of $y = k(x)$ and $y = q(x)$ in terms of $m$ and $n$ if $n$ is even.

Give your answer in the form $am + \frac{b}{n^2}$, where $a$ and $b$ are integers.

$$\text{Area} = 2\int_0^\pi (k(x) - q(x)) \, dx$$

$$= 4m + \frac{2\cos(n\pi) - 2}{n^2}$$

If $n$ is even $\cos(n\pi) = 1$.

$$= 4m + \frac{0}{n^2}$$

OR

By symmetry, when $n$ is even, $\int_0^\pi q(x) \, dx = 0$.

Therefore $\text{Area} = 2\int_0^\pi (k(x)) \, dx = 4m$
Question 4di

\[ 2 \int_0^\pi \left( m \cdot \sin(x) - \frac{1}{n} \cdot \sin(n \cdot x) \right) dx \]

\[ = 2 \cdot \left( 2 \cdot m \cdot n^2 + \cos(n \cdot \pi) - 1 \right) \]

\[ = 2 \cdot \left( 2 \cdot m \cdot n^2 + \cos(n \cdot \pi) - 1 \right) \bigg|_{n=2} \]

\[ = 4 \cdot m \]
ii. Find the area enclosed by the graphs of \( y = k(x) \) and \( y = q(x) \) in terms of \( m \) and \( n \) if \( n \) is odd.

Give your answer in the form \( am + \frac{b}{n^2} \), where \( a \) and \( b \) are integers.

\[
\text{Area} = 2 \int_0^\pi (k(x) - q(x)) \, dx
\]

\[
= 4m + \frac{2 \cos(n\pi) - 2}{n^2}
\]

If \( n \) is odd \( \cos(n\pi) = -1 \).

\[
= 4m - \frac{4}{n^2}
\]
Question 5 (15 marks)

a. Let \( S(t) = 2e^3 + 8e^{-\frac{2t}{3}} \), where \( 0 \leq t \leq 5 \).

i. Find \( S(0) \) and \( S(5) \).

\[
S(t) = 2e^3 + 8e^{-\frac{2t}{3}}
\]

\[
S(0) = 10, \quad S(5) = 2e^3 + 8e^{-\frac{10}{3}}
\]
Question 5aii

ii. The minimum value of $S$ occurs when $t = \log_e(c)$.

State the value of $c$ and the minimum value of $S$.

Solve $S'(t) = 0$ for $t$.

$t = 3 \log_e(2) = \log_e(8)$

$c = 8$

$S(\log_e(8)) = 6$
Question 5a(iii)

iii. On the axes below, sketch the graph of $S$ against $t$ for $0 \leq t \leq 5$. Label the end points and the minimum point with their coordinates.

![Graph of S against t]
iv. Find the value of the average rate of change of the function $S$ over the interval $[0, \log_e(c)]$.

Average rate of change = \[
\frac{S(\log_e(8)) - S(0)}{\log_e(8) - 0}
\]

= \[
\frac{6 - 10}{\log_e(8)}
\]

= \[-\frac{4}{\log_e(8)}\]
Question 5b

Let \( V : [0, 5] \to \mathbb{R}, \ V(t) = de^3 + (10 - d)e^{-\frac{2t}{3}}, \) where \( d \) is a real number and \( d \in (0, 10) \).

b. If the minimum value of the function occurs when \( t = \log_e(9) \), find the value of \( d \).

\[
V(t) = de^3 + (10 - d)e^{-\frac{2t}{3}}
\]

Solve \( V'(\log_e(9)) = 0 \) for \( d \).

\[
d = \frac{20}{11}
\]
c. i. Find the set of possible values of $d$ such that the minimum value of the function occurs when $t = 0$.

Solve $V'(0) = 0$ for $d$.

$$d = \frac{20}{3}$$

$$\frac{20}{3} \leq d < 10$$
Question 5ci

c.  i. Find the set of possible values of $d$ such that the minimum value of the function occurs when $t = 0$.

The graph shows that when $d = \frac{20}{3}$, the turning point minimum occurs at $t = 0$. For $\frac{20}{3} \leq d < 10$, there is an endpoint minimum at $t = 0$. 
Question 5cii

ii. Find the set of possible values of $d$ such that the minimum value of the function occurs when $t = 5$.

Solve $V'(5) = 0$ for $d$.

$$d = \frac{20}{2 + e^5}$$

$$0 < d \leq \frac{20}{2 + e^5}$$
ii. Find the set of possible values of $d$ such that the minimum value of the function occurs when $t = 5$.

The graph below shows that when $d = \frac{20}{2 + e^5}$, the turning point minimum occurs at $t = 5$.

For $0 < d \leq \frac{20}{2 + e^5}$, there is an endpoint minimum at $t = 5$. 
Question 5d

d. If the function $V$ has a local minimum $(a, m)$, where $0 \leq a \leq 5$, it can be shown that

$$m = \frac{k}{2} d^3 (10 - d)^{\frac{1}{3}}.$$ 

Find the value of $k$.

Solve $V'(a) = 0$ for $a$.

$$a = \log_e \left( \frac{20}{d} - 2 \right)$$

Solve $V \left( \log_e \left( \frac{20}{d} - 2 \right) \right) = \frac{k}{2} d^3 (10 - d)^{\frac{1}{3}}$ for $k$.

$$k = 3 \times 2^{\frac{1}{3}}$$

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d. If the function $V$ has a local minimum $(a, m)$, where $0 \leq a \leq 5$, it can be shown that

$$m = \frac{k}{2} \left( \frac{2}{3} \right)^{\frac{2}{3}} \left( 10 - \frac{20}{3} \right)^{\frac{1}{3}}.$$ 

Find the value of $k$.

OR

Use a value of $a$ where $0 \leq a \leq 5$.

Let $a = 0$.

$$V(0) = 10, \quad d = \frac{20}{3} \quad \text{from c.i.}$$

Solve $10 = \frac{k}{2} \left( \frac{20}{3} \right)^{\frac{2}{3}} \left( 10 - \frac{20}{3} \right)^{\frac{1}{3}}$ for $k$.

$$k = 3 \times 2^{\frac{1}{3}}$$